

A Black Hole Can Return a Qubit Before It Returns a Compass

Registered research scaffold — no numerical result

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Abstract

Hayden and Preskill showed that an old black hole returns a small, unstructured quantum system almost as soon as a matching sample of later radiation exists, once internal scrambling has run. This draft registers, without simulating it, what changes when the infalling system instead carries a heading tied to a conserved $U(1)$ or $SU(2)$ charge that the hole’s own evaporation must respect. Starting from a symmetry-covariant evaporation isometry, we state two differently shaped operational recovery problems evaluated on the same collected radiation: a logical threshold time $t_L(\epsilon)$ set by decoder fidelity and bounded in $[0, 1]$, and a frame threshold time $t_G(\delta)$ set by the quantum Cramér-Rao bound on estimating the heading and unbounded above. We prove, rather than assume, that the quantum Fisher information entering t_G equals $4 \text{Var}(\hat{J})/\hbar^2$ and is exactly constant along the covariant orbit; that covariance forces the evaporation isometry to be block-diagonal in total charge, with a sharper consequence for $SU(2)$: a covariant map can act freely only on the multiplicity label distinguishing repeated copies of a given total spin, never on the orientation degree of freedom inside one copy; and that a sector-respecting isometry conserves exactly the Hilbert-Schmidt norm of any inter-sector coherence block, so scrambling can only redistribute a fixed budget of frame information rather than create or destroy it. We derive and independently cross-check the $U(1)$ charge-sector dimension count $\binom{n}{k}$ and the $SU(2)$ Catalan-triangle multiplicity for up to ten elementary spins against a separate recursive Clebsch-Gordan computation, and use both to make precise why a charge-neutral qubit’s recovery is essentially unaffected by the symmetry while a charge-bearing heading’s is not. Two required exact limits are recovered: removing the charge, or granting an unlimited shared external reference frame, collapses the two thresholds together. A single toy accumulation law, evaluated exactly rather than simulated, reproduces the companion article’s

millisecond-scale qubit recovery time for a solar-mass hole and shows that the ordering of the compass threshold against it reverses sign depending only on a stated tolerance convention – an honestly reported instability in the headline claim, not a suppressed one. A trapped-ion experiment already validates the charge-blind half of this picture; nothing has yet tested the frame-recovery half. A symmetric random-circuit simulation with an optimized decoder and an optimized covariant frame estimator, and a proof that a charge-neutral qubit’s operative charge sector is typical rather than exceptional, remain future work, stated as such. A conducted priority search places the draft’s claimed contribution narrowly: that symmetry delays and partially obstructs Hayden-Preskill recovery is inherited from prior work, not reproduced here as new. **Keywords:** Hayden-Preskill recovery; black

hole information; quantum scrambling; superselection rules; quantum reference frames; quantum Fisher information; Cramér-Rao bound; Schur-Weyl duality; Page curve.

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Status: Draft scaffold. The symmetric random-circuit simulation, the optimized decoder and frame estimator, the typical-sector concentration proof, and external review are incomplete.

1 Setting the Comparison on One Evaporation Map

An old black hole, in Page’s technical sense, has already radiated more than half of its Bekenstein-Hawking entropy, so the early radiation R held by a distant observer is, on average, close to maximally entangled with whatever the hole’s interior still holds [5, 6]. Hayden and Preskill showed that once such a hole scrambles a further small system for about a scrambling time, a

modest additional sample of radiation suffices to reconstruct what fell in [1]. Every construction below lives in this regime; a young hole, with little in R , has neither recovery problem solved yet.

Two systems fall in together. The first, A , is a qubit prepared in an arbitrary state and coupled to no conserved quantity the hole tracks: $\hat{J}_A = 0$ identically, where $\hat{J}$ is the Hermitian generator of the conservation law under study. The second, C , is a compass: a carrier prepared in

$$|\psi(g)\rangle = e^{-ig\hat{J}/\hbar} |\psi(0)\rangle, \quad g \in [0, 2\pi),$$

a one-parameter $U(1)$ orbit generated by the same $\hat{J}$ the hole's own angular momentum, or other conserved charge, is built from. g is dimensionless and periodic; $\hat{J}$ carries units of action, so $g\hat{J}/\hbar$ is a dimensionless exponent, as required. Section 4 extends the same setup to a full $SU(2)$ heading, with $\hat{J}$ replaced by the triple $(\hat{J}_x, \hat{J}_y, \hat{J}_z)$ and g by a rotation $\Omega \in SU(2)$; the $U(1)$ case is used wherever a single scalar suffices, matching the restriction Peres and Scudo used for a single-axis carrier [10].

The hole's evaporation is modeled as an isometry $V : \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_{\text{rad}} \otimes \mathcal{H}_{\text{rem}}$, where $\mathcal{H}_{\text{in}}$ is the joint Hilbert space of the pre-evaporation interior together with A and C , $\mathcal{H}_{\text{rad}}$ is the space of radiation collected after infall, and $\mathcal{H}_{\text{rem}}$ is whatever interior remains. Conservation of the charge is imposed as an exact operator identity, for every g :

$$V U_{\text{in}}(g) = [U_{\text{rad}}(g) \otimes U_{\text{rem}}(g)] V. \quad (1)$$

Each $U(g)$ is a unitary representation of the same charge rotation on a different factor; none is an observable. Eq. (1) is not an extra postulate bolted onto Hayden-Preskill scrambling — it is the exact statement that the hole's internal dynamics cannot tell which heading was chosen before C fell in, once evaporation is written as a unitary (or isometric) process. It is the same setting Nakata, Wakakuwa, and Koashi analyzed for the case of a single conserved $U(1)$ charge on the input side [8], and the setting Tajima and Saito treat in full generality for an arbitrary compact Lie group [20]; Section 9 states precisely what those results already establish and what this draft adds to them.

Every clock reading below, t , is the distant, asymptotically flat coordinate time an outside observer's wristwatch registers while radiation accumulates — never the infall proper time, which is negligible on this scale, and never a local near-horizon proper time, which redshifts against t by the same factor entering the Hawking temperature [7]. To avoid overloading a single letter, the radiation collected up to time t beyond the fixed early set R is written B_t , and $R_t \equiv R \cup B_t$ denotes everything the recovering agent actually holds at t . The companion landing article uses the same growing batch under the label " C_t " while also using C for

the compass itself; the split into C and B_t here is a notational refinement for this more formal treatment and changes no claim below.

2 Two Different Shapes of “Given Back”

A 's recovery is a yes-or-no promise sharpened into a number. There exists a recovery channel $\mathcal{R}$, built from operations available to an agent holding R_t , whose entanglement fidelity to the identity channel on A is within ϵ of perfect:

$$\epsilon_L(t) = \min_{\mathcal{R}} [1 - F_e(\mathcal{R} \circ \mathcal{N}_t, \text{id})], \quad t_L(\epsilon) = \min\{t : \epsilon_L(t) \leq \epsilon\}. \quad (2)$$

$\mathcal{N}_t$ is the channel from A to R_t induced by scrambling; $\epsilon_L(t) \in [0, 1]$ is dimensionless, t_L has units of time.

C 's recovery cannot be phrased this way, because g is continuous and no channel reproduces an unknown continuous value exactly. The honest promise is metrological: the best available covariant measurement and estimator $\hat{g}$ built from R_t satisfies

$$\text{Var } \hat{g} \geq \frac{1}{F_Q[\rho_{\text{rad}}(g)]}, \quad t_G(\delta) = \min\{t : \text{Var } \hat{g}|_{R_t} \leq \delta^2\}. \quad (3)$$

F_Q is the quantum Fisher information of the radiation's reduced state with respect to g [12]; since g is dimensionless, so is F_Q . δ is the observer's stated alignment tolerance, and t_G has units of time.

Proposition 2.1 (Fisher information of a covariant orbit). *For $|\psi(g)\rangle = e^{-ig\hat{J}/\hbar} |\psi(0)\rangle$ with $\hat{J}$ Hermitian, the quantum Fisher information of the pure-state family with respect to g is exactly*

$$F_Q = 4 \left[\langle \partial_g \psi | \partial_g \psi \rangle - |\langle \psi | \partial_g \psi \rangle|^2 \right] = \frac{4}{\hbar^2} \text{Var}(\hat{J}), \quad (4)$$

evaluated at any g , with $\text{Var}(\hat{J})$ taken in $|\psi(g)\rangle$.

Proof. $\partial_g |\psi(g)\rangle = -\frac{i}{\hbar} \hat{J} |\psi(g)\rangle$. Then $\langle \partial_g \psi | \partial_g \psi \rangle = \langle \psi | \hat{J}^2 | \psi \rangle / \hbar^2$ since $\hat{J}$ is Hermitian, and $\langle \psi | \partial_g \psi \rangle = -i \langle \hat{J} \rangle / \hbar$, so $|\langle \psi | \partial_g \psi \rangle|^2 = \langle \hat{J} \rangle^2 / \hbar^2$. Substituting into the defining expression for F_Q gives Eq. (4). $\square$

Proposition 2.2 (g -independence along the orbit). *$\text{Var}(\hat{J})$, and hence F_Q , is exactly the same at every point of the orbit $\{|\psi(g)\rangle\}_{g \in [0, 2\pi)}$.*

Proof. $\langle \psi(g) | \hat{J}^k | \psi(g) \rangle = \langle \psi(0) | e^{ig\hat{J}/\hbar} \hat{J}^k e^{-ig\hat{J}/\hbar} | \psi(0) \rangle = \langle \psi(0) | \hat{J}^k | \psi(0) \rangle$ for $k = 1, 2$, because $\hat{J}$ commutes with any function of itself. Hence $\text{Var}(\hat{J})$ is g -independent, and so is F_Q by Proposition 2.1. $\square$

Both propositions are cross-checked numerically in `check_qubit_compass_recovery.py`: a finite-difference evaluation of F_Q at an explicit state matches the closed form of Eq. (4) to eight significant figures, and the finite-difference value is reproduced at five separate base points g_0 to the same precision, confirming Proposition 2.2 is not an artifact of the specific g chosen. The same conclusion is used implicitly by the companion article and by the storage-ring metrology literature more generally [13], but is stated and proved here rather than assumed.

The shape difference between Eqs. (2) and (3) is the entire reason a comparison needs registering rather than being read off by inspection. $\epsilon_L(t)$ is bounded in $[0, 1]$ and, once decoupling sets in, can approach its floor abruptly. $\text{Var } \hat{g}$ has no upper bound and falls only as fast as F_Q grows. A single number "has it come back yet" therefore depends on which of two structurally different questions is being asked, and Section 7 shows the comparison additionally depends on a convention neither definition supplies on its own.

3 Schur Blocks: Why the Evaporation Map Cannot Mix Charge

Eq. (1) is usually read qualitatively, as "the hole conserves the charge." It has a sharper, provable consequence for the structure of V itself.

Proposition 3.1 (Covariance forces block-diagonality). *Let $\hat{J}_{\text{in}}$ generate $U_{\text{in}}(g) = e^{-ig\hat{J}_{\text{in}}/\hbar}$ and let $\hat{J}_{\text{out}} \equiv \hat{J}_{\text{rad}} \otimes \mathbb{K} + \mathbb{K} \otimes \hat{J}_{\text{rem}}$ generate $U_{\text{rad}}(g) \otimes U_{\text{rem}}(g)$. If V satisfies Eq. (1) for all g , then*

$$V\hat{J}_{\text{in}} = \hat{J}_{\text{out}}V. \quad (5)$$

Consequently, if $\hat{J}_{\text{in}}|\phi\rangle = q\hbar|\phi\rangle$, then $\hat{J}_{\text{out}}V|\phi\rangle = q\hbar V|\phi\rangle$: V maps the charge- q eigenspace of the input entirely into the charge- q eigenspace of the output, for every q .

Proof. Differentiate Eq. (1) with respect to g at $g = 0$. The left side gives $V \cdot (-i\hat{J}_{\text{in}}/\hbar)$; the right side gives $(-i\hat{J}_{\text{out}}/\hbar) \cdot V$, using $U_{\text{rad}}(0) = U_{\text{rem}}(0) = \mathbb{K}$. Equating and cancelling $-i/\hbar$ gives Eq. (5). The eigenspace statement follows immediately by applying both sides to $|\phi\rangle$. $\square$

This is not an extra assumption: it is an algebraic consequence of imposing exact covariance for every g , not only for $g = 0$. In the $U(1)$ case, writing Π_q for the projector onto charge q (input side) and Π'_q for the same label on the output side, Proposition 3.1 says $V = \sum_q \Pi'_q V \Pi_q$: V is exactly block-diagonal in q , with $\sum_q \Pi_q = \mathbb{K}_{\mathcal{H}_{\text{in}}}$ providing a resolution of the identity that Section 4 counts explicitly.

Block-diagonality alone does not say how V acts *within* a block. The paper's working model — stated as an ansatz, not derived from any specific microscopic evaporation Hamiltonian — is that each block acts as an independent Haar-random isometry, consistent with the idea that the hole scrambles as thoroughly as the conservation law permits. This is the finite-dimensional object `check_qubit_compass_recovery.py` builds and tests directly: for $n = 2, 3, 4$ elementary charge units it assembles a block-diagonal unitary from independently Haar-sampled blocks and confirms, to machine precision, both that the assembled matrix is unitary and that it commutes exactly with $e^{-ig\hat{J}}$ for random g — i.e., that the construction is a faithful finite-size instance of Eq. (1), not merely a plausible cartoon of it.

For $SU(2)$, the same differentiation argument applied to each of $\hat{J}_x, \hat{J}_y, \hat{J}_z$ gives block-diagonality in total spin j rather than in a single number q , but the conclusion is sharper. Decompose $\mathcal{H}_{\text{in}} = \bigoplus_j \text{Mult}_j \otimes V_j$, where V_j is the $(2j + 1)$ -dimensional carrier of the spin- j irreducible representation and Mult_j is the multiplicity space distinguishing repeated copies of the same j , and similarly for the output. Schur's lemma for intertwining maps between isomorphic representations forces

$$V = \bigoplus_j M_j \otimes \mathbb{K}_{V_j}, \quad (6)$$

where $M_j : \text{Mult}_j \rightarrow \text{Mult}'_j$ is an arbitrary linear map and $\mathbb{K}_{V_j}$ is the identity on the spin carrier itself. Eq. (6) says a covariant evaporation map has *no freedom at all* to act on the orientation degree of freedom inside a fixed copy of a spin- j representation; every bit of scrambling freedom available to it lives in M_j , i.e., in which physical copy, among the $\dim \text{Mult}_j$ available, a given orientation ends up embedded in. This is the structural fact behind Section 6: the heading itself is never scrambled by a covariant map, only *which multiplicity slot* carries it, and recovering the heading from radiation is really the problem of locating the right slot.

4 Counting the Sectors: $U(1)$ Binomials and $SU(2)$'s Catalan Triangle

Both known limits in Section 7 and the qubit-versus-compass asymmetry itself depend on how large the charge sectors of Proposition 3.1 actually are. This section derives and checks the two counts the rest of the paper uses.

U(1). Model the charge degree of freedom of the interior-plus-infalling system as n elementary two-level units, each contributing 0 or 1 to the total charge (a

spin chain in the $\hat{J}_z$ basis is the standard example). The charge- k sector is spanned by all length- n bit strings with exactly k ones, so

$$d(n, k) = \binom{n}{k}, \quad \sum_{k=0}^n d(n, k) = 2^n. \quad (7)$$

The sum rule is the resolution of the identity $\sum_k \Pi_k = \mathbb{1}$ made numerical; `check_qubit_compass_recovery.py` confirms it exactly for $n = 1$ through 12. For $n = 4$, the sector dimensions are 1, 4, 6, 4, 1 (checked directly by hand: $\binom{4}{0} = 1$, $\binom{4}{1} = 4$, $\binom{4}{2} = 6$).

SU(2). For n spin- $\frac{1}{2}$ constituents, the spin content is fixed by iterating the elementary branching rule $j \otimes \frac{1}{2} = (j + \frac{1}{2}) \oplus (j - \frac{1}{2})$. Writing $m(n, j)$ for the multiplicity of spin j in the n -fold product,

$$m(n, j) = m(n-1, j-\frac{1}{2}) + m(n-1, j+\frac{1}{2}), \quad (8)$$

with $m(0, 0) = 1$ and $m(0, j) = 0$ otherwise. The closed form solving this recursion is the Catalan-triangle formula

$$m(n, j) = \binom{n}{n/2-j} - \binom{n}{n/2-j-1}, \quad (9)$$

understood as zero whenever $n/2 - j$ is not a non-negative integer.

Proposition 4.1 (Eq. (9) solves the recursion). *The right-hand side of Eq. (9) satisfies Eq. (8) and the boundary condition at $n = 0$.*

Proof. Write $k \equiv n/2 - j$, an integer when $n, 2j$ share parity. The claim is $\binom{n}{k} - \binom{n}{k-1} = [\binom{n-1}{k} - \binom{n-1}{k-1}] + [\binom{n-1}{k-1} - \binom{n-1}{k-2}]$, i.e. $\binom{n}{k} - \binom{n}{k-1} = \binom{n-1}{k} - \binom{n-1}{k-2}$. Using Pascal's rule $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ twice on the left side gives $\binom{n-1}{k} + \binom{n-1}{k-1} - \binom{n-1}{k-1} - \binom{n-1}{k-2} = \binom{n-1}{k} - \binom{n-1}{k-2}$, matching the right side exactly. At $n = 0$, $k = 0$ (i.e. $j = 0$) gives $\binom{0}{0} - \binom{0}{-1} = 1 - 0 = 1$, and every other j gives 0, matching $m(0, \cdot)$. $\square$

Hand-checked cases, verified independently against the recursion in `check_qubit_compass_recovery.py`: for $n = 2$, one singlet ($j = 0$) and one triplet ($j = 1$), dimensions $1 + 3 = 4 = 2^2$; for $n = 3$, two doublets ($j = \frac{1}{2}$) and one quadruplet ($j = \frac{3}{2}$), $2 \times 2 + 4 = 8 = 2^3$; for $n = 4$, two singlets, three triplets, one quintuplet, $2 \times 1 + 3 \times 3 + 1 \times 5 = 16 = 2^4$. The self-check program reproduces the recursion independently by repeated branching (not by evaluating Eq. (9) and checking it against itself) for n up to ten and confirms $\sum_j (2j+1) m(n, j) = 2^n$ in every case, the SU(2) analogue of the U(1) sum rule above.

These counts do the following work later in the paper. In Section 5, a charge-neutral A 's recovery is controlled by a single sector of the joint system, whose dimension

— via Eq. (7) or (9) — differs from the total dimension only by a sub-extensive, typically logarithmic correction once the sector is a typical one. In Section 6, the SU(2) multiplicity $m(n, j)$ is exactly the number of slots Eq. (6)'s map M_j can shuffle a heading between, so $\ln m(n, j)$ sets the scale of "how much radiation" is needed before a covariant estimator can tell which slot the heading actually landed in.

5 Recovering the Qubit: Decoupling Blind to Charge

Because A carries no charge, its presence changes nothing about which sector the joint system (interior + C) occupies: the total charge label of $\mathcal{H}_{\text{in}}$ is inherited entirely from the pair (interior, C), and Proposition 3.1 places V 's action on A 's degrees of freedom entirely *inside* whichever sector that pair happens to occupy. Conditioned on a fixed sector of dimension d_q , A 's decoupling problem is then exactly the ordinary, unconstrained Hayden-Preskill counting argument run on an environment of size d_q rather than the full interior dimension d_{BH} : collecting a handful more qubits of new radiation than were thrown in suppresses ϵ_L exponentially in that handful, provided d_q is large [1, 15].

This reduction is exact as stated; what it leaves open is whether the *actual* sector occupied is typical. If the charge carried by C and the interior is distributed thermally over a large number of near-degenerate microstates, standard large-deviation reasoning says the occupied sector's dimension satisfies $\ln d_q = \ln d_{\text{BH}} - O(\text{sub-extensive})$ with overwhelming probability, so t_L computed from d_q and from d_{BH} agree to leading order. This concentration claim is stated as an assumption here, not proved; §10 registers its proof as outstanding. Granting it, t_L depends on the symmetry constraint only through a correction the constraint itself does not control, which is the precise sense in which "removing the conservation law" and "the qubit already does not feel it" are two descriptions of the same fact.

What sets the scale of t_L under this reduction is the same fast-scrambling estimate used without a charge constraint. Maldacena, Shenker, and Stanford's bound on the growth rate of an out-of-time-order correlator, $\lambda_L \leq 2\pi/\beta$ at inverse temperature β and saturated by a horizon, combined with Sekino and Susskind's conjecture that black holes scramble across all their degrees of freedom in a time growing only logarithmically with entropy [2, 3], gives

$$t_* \sim \frac{\beta}{2\pi} \ln S, \quad \beta = \frac{\hbar}{k_B T_H} = \frac{8\pi G M}{c^3}, \quad (10)$$

with $S = A/4\ell_P^2$ the Bekenstein-Hawking entropy in units of k_B and the second equality following from

Hawking's temperature formula for a Schwarzschild hole [6, 7]. Evaluated exactly for a solar mass, $\beta \approx 1.238 \times 10^{-4} \text{ s}$, $\ln S \approx 177.3$, and $t_* \approx 3.49 \text{ ms}$ — an arithmetic evaluation of Eq. (10), reproduced in `check_qubit_compass_recovery.py`, not a new prediction. A real hole's greybody factors change the effective emission rate by an order-one-to-few factor without changing the logarithmic scaling in S that makes t_* short compared to the hole's own evaporation time [2]; the millisecond figure should be read to that precision, not to the digits displayed.

Page's exact formula for the average entanglement entropy of an m -dimensional subsystem of an mn -dimensional Haar-random pure state,

$$\langle S_{m,n} \rangle = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}, \quad (11)$$

gives, at $m = 2$, the exact values $\langle S_{2,4} \rangle = 0.50952$, $\langle S_{2,16} \rangle = 0.64652$, $\langle S_{2,64} \rangle = 0.68144$ nats, closing in on $\ln 2 = 0.69315$ with the residual gap shrinking by a factor of 3.94 and then 3.98 as n quadruples [4]. This is an exact, hand-checkable corroboration of "each further collected qubit roughly halves what remains to recover," not a re-derivation of Hayden and Preskill's bound, and it is why $t_L(\epsilon)$ depends on ϵ only logarithmically. None of this requires the hole's true dynamics to be exactly Haar-random for unbounded time: Harrow and Low showed circuits of polynomial depth already approximate the first two moments of the Haar measure closely enough for arguments of this kind to apply [14], and Section 10 states precisely why the charge-conserving analogue of that statement — that a polynomial-depth symmetric circuit approximates a symmetric unitary design, not merely an unconstrained one — is assumed here rather than established.

6 Why the Compass's Ledger Needs a Different Meter

C 's heading is not stored in any single sector; it is stored, by construction, in the relative phase and amplitude *between* sectors — in $\text{SU}(2)$ language, in which orientation within a multiplicity copy the compass points to, per Eq. (6). Bartlett, Rudolph, and Spekkens call this kind of fact unspeakable: no classical message can convey it, because "north" has no meaning without a physical system that already points north to compare against [9]. The quantity F_Q of Proposition 2.1 is precisely a measure of how much of this unspeakable content the compass's own preparation carries.

Lemma 6.1 (Coherence is redistributed, not created). *Let $V : \mathcal{H} \rightarrow \mathcal{H}'$ be an isometry ($V^\dagger V = \mathbb{I}_{\mathcal{H}}$) that is block-diagonal with respect to a sector decomposition on*

both sides, $V = \sum_q \Pi'_q V \Pi_q$. For any operator ξ on $\mathcal{H}$ and any pair of sectors $q \neq q'$, writing $\xi_{qq'} \equiv \Pi_q \xi \Pi_{q'}$,

$$(V \xi V^\dagger)_{qq'} = (\Pi'_q V \Pi_q) \xi_{qq'} (\Pi'_{q'} V \Pi_{q'})^\dagger,$$

and its Hilbert-Schmidt norm equals $\|\xi_{qq'}\|_{\text{HS}}$ exactly.

Proof. Write $A_q \equiv \Pi'_q V \Pi_q$, so $(V \xi V^\dagger)_{qq'} = A_q \xi_{qq'} A_{q'}^\dagger$ by block-diagonality (cross terms with $q_1 \neq q$ or $q_2 \neq q'$ vanish since $\Pi'_q V \Pi_{q_1} = 0$ unless $q_1 = q$). From $V^\dagger V = \mathbb{I}_{\mathcal{H}}$ and block-diagonality, $\sum_q \Pi_q V^\dagger \Pi'_q V \Pi_q = \mathbb{I}_{\mathcal{H}}$ term by term, so $A_q^\dagger A_q = \Pi_q$ for every q ; each A_q is an isometry on its sector. Then

$$\|A_q \xi_{qq'} A_{q'}^\dagger\|_{\text{HS}}^2 = \text{Tr} [A_{q'}^\dagger \xi_{qq'}^\dagger A_q^\dagger A_q \xi_{qq'} A_{q'}^\dagger] = \text{Tr} [\xi_{qq'}^\dagger \xi_{qq'} A_{q'}^\dagger A_{q'}],$$

using $A_q^\dagger A_q = \Pi_q$, $\Pi_q \xi_{qq'} = \xi_{qq'}$, and cyclicity of the trace. Substituting $A_{q'}^\dagger A_{q'} = \Pi_{q'}$ and $\xi_{qq'} \Pi_{q'} = \xi_{qq'}$ gives $\text{Tr} [\xi_{qq'}^\dagger \xi_{qq'}] = \|\xi_{qq'}\|_{\text{HS}}^2$. $\square$

Lemma 6.1 is checked numerically in `check_qubit_compass_recovery.py`, not on a generic isometry alone but on an explicit block-diagonal one: a 3-charge-unit input (sector dimensions 1, 3, 3, 1) isometrically embedded into arbitrarily larger, same-label output sectors, with a random coherence block placed between two distinct input sectors. The output block's Hilbert-Schmidt norm matches the input block's to nine significant figures, and every other sector-pair block of the output receives exactly zero coherence, confirming both halves of the lemma rather than only the norm identity in isolation. Its content: a charge-respecting evaporation map can move a fixed budget of inter-sector coherence to a different pair of physical carriers, but the raw magnitude of that budget is conserved by V itself. Bartlett, Rudolph, and Spekkens's asymmetry monotone strengthens this from an equality under an isometry to an inequality under the further, physically unavoidable step of discarding (tracing out) whatever part of the output is not collected: asymmetry cannot increase under any operation that respects the symmetry, including a partial trace [9]. The qubit has no analogous quantity, because a charge-neutral system carries no inter-sector coherence to begin with; this is the precise, checkable sense in which A and C are not two instances of one problem.

What the fixed budget looks like once diluted across the joint interior-plus-radiation system, though, is not computed by Lemma 6.1 alone — that requires the actual reduced dynamics, which this draft does not simulate. In its place, following the companion article, take a stated phenomenological ansatz: each of the $N_\gamma(t) \sim t/\beta$ radiation quanta collected by t contributes an independent, per-quantum share f_1 of Fisher information about g , the standard-quantum-limit scaling appropriate when probes are used independently rather

than coherently [13]:

$$F_Q(t) \approx f_1 \frac{t}{\beta}, \quad t_G(\delta) \approx \frac{\beta}{f_1 \delta^2}. \quad (12)$$

This model has an explicit ceiling built in, $F_Q(t) \leq F_Q^{(0)} = 4 \text{Var}(\hat{J})/\hbar^2$ by Proposition 2.1 applied to C 's own initial preparation: no amount of radiation reveals more about g than the compass ever carried. A tolerance tighter than $1/\sqrt{F_Q^{(0)}}$ is a request for better precision than C itself possessed, not a slower recovery. Within that ceiling, Kitaev, Mayers, and Preskill's simulation theorem is the sharpest known escape route: if the two parties already share an unlimited external reference, the superselection restriction can be entirely simulated away, and the alignment problem this section describes disappears [11]. In the $\text{SU}(2)$ language of Section 3, an external reference is exactly a device that already knows which multiplicity slot to look in; without one, that slot has to be found from the radiation itself, at the rate Eq. (12) states as an ansatz.

7 The Threshold Gap and the Limits It Must Reproduce

Collect the two thresholds into one signed, unit-checked quantity,

$$\Delta t_{LG}(\epsilon, \delta) \equiv t_G(\delta) - t_L(\epsilon), \quad (13)$$

measured in seconds and invariant under the group action because both terms are (Proposition 2.2 for t_G ; t_L never referred to g at all).

Proposition 7.1 (Trivial-symmetry limit). *If $\hat{J} = 0$, then $\Delta t_{LG} \rightarrow 0$ for every ϵ, δ .*

Proof. $\hat{J} = 0$ removes every eigenspace structure from Proposition 3.1: there is only one sector, and any continuous label C might still carry is encoded in radiation exactly as A 's logical content is, with no unspeakable component (Section 6 does not apply, since there is no non-trivial charge basis to be unspeakable relative to). The recovery problems for A and C become the same problem, so $t_G \rightarrow t_L$. $\square$

Proposition 7.2 (Shared-frame limit). *If the two parties additionally share an unlimited, pre-established external reference for $\hat{J}$, then $\Delta t_{LG} \rightarrow 0$ for every ϵ, δ , independent of $\hat{J} \neq 0$.*

Proof. Kitaev, Mayers, and Preskill's simulation theorem states that a superselection rule restricting operations to those commuting with a group can be entirely simulated away once the parties share a suitable reference system [11]. An unlimited shared reference removes the restriction that made Section 6's argument bite: the recovering agent may act as though no conservation law constrained V at all, and the argument of

Proposition 7.1 applies to the effective, unconstrained problem. $\square$

Both limits are required checks, not optional flourishes: a construction that failed to collapse to ordinary Hayden-Preskill recovery in either would not be trustworthy where the two thresholds actually differ. A third, trivial floor closes the set: for B_t smaller than the infalling systems themselves, or before scrambling has had time to run, neither $\epsilon_L(t)$ nor $\text{Var} \hat{g}$ meets any nontrivial bound, which is causality, not a result.

A worked, explicitly labelled toy evaluation. Taking the solar-mass numbers of Section 5, $t_L \approx 3.49$ ms, i.e. $t_L/\beta \approx 28.2$. Using Eq. (12) with the illustrative, explicitly stated choice $f_1 = 1$: a tolerance $\delta = 0.1$ rad ($\approx 6^\circ$) gives $t_G/\beta = 100$, so $t_G \approx 12.4$ ms and $\Delta t_{LG} \approx +8.9$ ms, matching the title's ordering. A coarser tolerance, $\delta = 0.5$ rad ($\approx 29^\circ$), gives $t_G/\beta = 4$, so $t_G \approx 0.50$ ms and $\Delta t_{LG} \approx -3.0$ ms: on the identical hole and model, the compass now returns first. The crossover, at $1/(f_1 \delta^2) = \ln S/2\pi$, sits at $\delta^* \approx 0.188$ rad $\approx 10.8^\circ$. Every number in this paragraph is an exact evaluation of Eqs. (10) and (12), reproduced by `check_qubit_compass_recovery.py`; none is a simulated or measured quantity.

This reversal is the kill criterion in action, not a defect papered over. Δt_{LG} does not vanish under fair resource matching — both thresholds draw on the same radiation, collected by the same observer, under the same scrambling — so it survives that half of the kill test. It does, however, change sign under a joint choice of ϵ, δ this construction has no privileged way to fix: a fidelity and a normalized angular tolerance measure different things, and nothing forces a "demanding" choice of one onto the same point of stringency as a "demanding" choice of the other. The correct statement is conditional — for comparably stringent, no-external-frame tolerances on both sides, the qubit returns first — and any presentation dropping that qualifier overclaims what Eqs. (13)–(12) actually establish. Whether Δt_{LG} additionally reduces to a decoder artifact under an *optimized* decoder, rather than the counting-argument estimate of Section 5, is the other half of the kill test, and is not settled here; Section 10 registers it as outstanding.

8 What a Trapped-Ion Circuit Already Shows, and What It Cannot

Landsman and colleagues built a seven-qubit ion-trap circuit engineered to scramble like the toy models this literature is built on, then used a conditional teleportation protocol from the same family as Yoshida and Kitaev's efficient decoder to test whether information injected into the circuit could be recovered from its

output exactly when the internal dynamics were genuinely scrambling [15, 16]. Teleportation fidelity rose sharply above the classical baseline in the scrambling regime and did not in a matched non-scrambling control: a real, non-gravitational confirmation that the mechanism behind t_L is a physical effect, not an artifact of an idealized counting argument.

In the language of Sections 3–7, that circuit has $\hat{J} = 0$ on every qubit involved: there is no conserved charge for V to respect, so Proposition 7.1 applies verbatim, and the experiment is an empirical instance of the trivial-symmetry limit rather than a test of anything Section 6 describes. Its seven qubits carry no analogue of C , no heading was thrown in, and no alignment problem of the kind Bartlett, Rudolph, and Spekkens formalize was ever posed to the circuit [9]. The honest parameter range this draft can claim is therefore narrow and asymmetric by construction: the charge-blind mechanism behind t_L has a laboratory analogue at the scale of a handful of trapped ions, backed by a measured fidelity; the mechanism behind t_G has none. Every number attached to t_G in Section 7 — the accumulation ansatz, f_1 , δ^* — is a phenomenological construction consistent with the rigorous limits of Section 7, not a measured or simulated quantity, and should be read that way until an experiment analogous to Landsman et al.’s exists for a genuinely charge-carrying probe.

9 Where This Sits Relative to the Symmetry-Constrained Recovery Literature

The qualitative claim that a conservation law delays and partially obstructs Hayden-Preskill recovery is not new, and this paper does not present it as new. Nakata, Wakakuwa, and Koashi proved it directly for a $U(1)$ charge, exhibiting both a delay characterized by the thermodynamics of the conserved quantity and an information remnant tied to how strongly the initial state breaks the symmetry [8]. Tajima and Saito proved a general version for an arbitrary compact Lie group, framing the obstruction as a trade-off between symmetry and quantum coherence and connecting it to the approximate Eastin-Knill and Wigner-Araki-Yanase theorems [20]. Chen and Tang computed an explicit transition time at which the entanglement asymmetry of Hayden-Preskill radiation vanishes under an emergent $U(1)$ symmetry [21]. Tang, Vardhan, and Wang used quantum Fisher information to bound how precisely an agent can estimate elapsed time itself in a chaotic system, including a black hole, from measurements on a subsystem [22]. Any of these four papers, on their own, is closer prior art for "symmetry limits recovery" than anything this draft adds; a reader who wants that result should read them, not this scaffold.

What this draft’s search — targeted arXiv and journal queries against the terms "Hayden-Preskill," "symmetry," "reference frame," "Fisher information," and "recovery time," not an exhaustive systematic review — did not turn up is a construction that places $t_L(\epsilon)$ and $t_G(\delta)$ side by side as two differently *shaped* operational objects (one fidelity-bounded, one Cramér-Rao-unbounded) and reports their signed difference as a quantity that changes sign under a stated, examined convention. Chen and Tang’s transition time characterizes when the radiation’s asymmetry itself vanishes, a single threshold rather than a comparison between two tasks; Tang, Vardhan, and Wang’s Fisher information targets estimating time, not a conserved heading; Tajima and Saito’s trade-off is a general inequality between recovery error and a coherence cost, not two named clock readings compared for a specific hole. If this narrow difference in framing is what survives a fuller search, this draft’s residual, honestly scoped contribution is: the explicit dual-threshold comparison itself; the Schur-block derivation (Proposition 3.1 and Eq. (6)) making precise which degree of freedom a covariant map can and cannot touch; the $U(1)/SU(2)$ sector-counting ledger of Section 4, cross-checked independently rather than asserted; and Lemma 6.1’s exact coherence-conservation statement. None of these four items has been found elsewhere by the search actually performed, but that search is registered here as partial, and Section 10 lists a completed priority search as an outstanding gate rather than a settled fact.

The random-circuit literature with Abelian conservation laws — Khemani, Vishwanath, and Huse on operator spreading with a diffusive conserved density; Rakovszky, Pollmann, and von Keyserlingk on the resulting hydrodynamics of out-of-time-order correlators; and Agrawal et al. on the charge-sharpening transition separating regimes where measurements can or cannot efficiently learn a circuit’s total charge — supplies the closest existing evidence for or against the block-Haar-random ansatz of Section 3, without itself being framed as a black-hole recovery problem [17–19]. In particular, the charge-sharpening transition is a direct, already-published warning against this draft’s ansatz: it shows that whether a fixed amount of measurement can learn a circuit’s total charge efficiently is itself a sharp, non-trivial question with a phase transition in it, not a foregone consequence of charge conservation alone. Reconciling Section 6’s toy accumulation law with that transition, rather than simply asserting an SQL-type growth law, is registered as unresolved in Section 10.

10 The Registration Gate: What Blocks This From Being a Results Paper

Everything computed in this draft is an exact combinatorial identity, an exact algebraic identity, a finite-precision structural check, or an arithmetic evaluation of a previously published closed-form formula. Nothing in it is the symmetric random-circuit simulation named in Section 9's framing of this project, and this section states, without qualification, what stands between the two.

The first gate is the simulation itself. No charge- or spin-conserving random-circuit ensemble has been implemented beyond the finite-size, purely structural block-diagonal construction in `check_qubit_compass_recovery.py`, which verifies that such an ensemble is unitary and covariant, not that it reproduces real scrambling dynamics. Until it exists, the numerical result named in this project's own registration — exact symmetric random-circuit simulations with optimized logical recovery and frame estimation — has not been produced, and every sentence in Sections 5–7 that quotes a specific value of t_L , t_G , or Δt_{LG} is a closed-form illustration, not a simulated measurement.

The second and third gates are the two optimizations the framework was built to compare. Section 5's estimate of t_L uses the counting argument behind Hayden and Preskill's original bound, not an explicitly constructed and optimized decoding circuit of the kind Yoshida and Kitaev supply for the unconstrained case [15]; a symmetric analogue of their decoder has not been built here. Section 6's estimate of t_G uses the phenomenological standard-quantum-limit ansatz of Eq. (12), not a constructed covariant measurement approaching the quantum Cramér-Rao bound of Eq. (3). Both gates block the same sentence: nothing in this draft has verified that t_L and t_G , as actually achievable by the best available protocols rather than by a counting argument and an ansatz, behave as Sections 5–7 assume.

The fourth gate is the typical-sector concentration claim in Section 5: that the charge sector occupied by a real interior-plus-compass system is close in dimension to a typical sector, so that A 's recovery time tracks the unconstrained estimate to leading order. This is stated as an assumption, not derived from any specific interior microstate distribution, and it is the sentence in Section 5 claiming " t_L depends on the symmetry constraint only through a correction the constraint itself does not control" that remains unproved until a concentration bound is established.

The fifth gate is the reconciliation Section 9 flags but does not close: Agrawal et al.'s charge-sharpening

transition shows that whether measurements can efficiently learn a circuit's total charge is a sharp, nontrivial question, not automatic from charge conservation [17]. Section 6's accumulation law, Eq. (12), has not been checked against that transition, and if the two are in tension, the numerical values quoted in Section 7 — $\delta^* \approx 0.188$ rad and the specific reversal at $\delta = 0.5$ rad — would need to be recomputed rather than merely re-derived with a different f_1 .

The sixth gate is the priority search itself. Section 9 registers targeted queries, not an exhaustive literature review; the specific claim "this draft's dual-threshold framing was not found elsewhere" is only as strong as that search, and a fuller one is a precondition for asserting novelty in any venue narrower than this registered scaffold.

The seventh gate is scope: the worked numerical illustration in Section 7 is carried out only for the U(1) case. The SU(2) multiplicity machinery of Section 4 and the orientation/multiplicity split of Section 3 have not been connected to an analogous worked crossover number, so no claim here extends the specific millisecond-scale reversal to a genuinely three-axis heading.

The eighth gate is external review by a named subject-matter reviewer with expertise in black-hole information, quantum reference frames, and random-circuit thermalization with symmetry, who has not yet been approached. Concretely, before any statement in this draft is reported as a finding rather than a scaffold, all eight gates above must close: a validated symmetric-circuit simulation; an optimized symmetric decoder and an optimized covariant estimator; a proved typical-sector concentration bound; a reconciliation with the charge-sharpening literature; a completed priority search; an SU(2) worked illustration; and external review. As of this draft, all eight are open, and any one of them being open is sufficient reason to withhold a numerical claim.

11 What a Comparison of Two Recovery Clocks Is Actually Worth

Sorted by what actually supports them: that an old hole's radiation is close to maximally entangled with its remaining interior, that scrambling proceeds no faster than a chaos bound saturated by horizons, and that a small system thrown in becomes recoverable from a matching sample of later radiation are inherited baselines, owed to Page, Hayden and Preskill, Sekino and Susskind, and Maldacena, Shenker, and Stanford [1–3, 5]. That a global symmetry delays and partially obstructs that recovery is likewise inherited, from Nakata, Wakakuwa, and Koashi and from Tajima and Saito's more general treatment [8, 20]. What belongs to this

draft is narrower: the explicit pairing of a fidelity-bounded $t_L(\epsilon)$ and a Cramér-Rao-unbounded $t_G(\delta)$ as two structurally different promises evaluated on the same radiation; the proof, not assumption, that covariance forces block-diagonality in charge and, for $SU(2)$, confines a covariant map's freedom to the multiplicity label alone; the independently cross-checked $U(1)$ and $SU(2)$ sector counts; the exact coherence-conservation lemma; and the worked, explicitly labelled toy evaluation that reproduces the qubit's millisecond-scale return and shows the compass's ordering against it is not fixed, only conditional.

The kill criterion stated at the outset — that the delay must not vanish under fair resource matching or reduce entirely to a decoder artifact — is only half addressed. The delay does not vanish under resource matching: both required limits (Propositions 7.1 and 7.2) collapse the two thresholds together exactly as an honest extension of Hayden-Preskill recovery must, and the toy model's reversal at loose tolerance is reported rather than concealed. Whether the delay is entirely a decoder artifact — an accident of using a counting-argument estimate for t_L rather than an actually optimized symmetric decoder — is not settled, and Section 10 names this as the sharpest open gate rather than a closed question dressed up as one.

What survives, narrowly, is a structural fact rather than a general theorem: a black hole does not have one intrinsic opinion about whether a generic bit or a charge-bearing heading comes back first. It has one mechanism, scrambling constrained by a conservation law, and Sections 3 and 4 show exactly which part of that mechanism a symmetry can and cannot touch — the multiplicity label, freely; the orientation inside a fixed representation, never. Which of the two return times is smaller is then a fact about a chosen pair of tolerances, not about horizons in general. The Page curve already guarantees, at the level of von Neumann entropy, that unitary evaporation eventually returns everything [4, 5]. It does not say that "everything" arrives on one schedule, and a future account reporting a single recovery time for all thrown-in information would be quietly averaging over exactly the distinction t_L and t_G are built to keep apart — a distinction this draft has registered, checked wherever an exact check was available, and otherwise left visibly open rather than filled in by assumption.

A Deriving and Verifying the $SU(2)$ Catalan-Triangle Multiplicity

Section 4 proves Eq. (9) by showing it solves the branching recursion Eq. (8). This appendix gives a second, independent derivation, by telescoping the $U(1)$ sector

counts of Eq. (7) themselves, and records the numerical cross-check.

For n spin- $\frac{1}{2}$ constituents, identify the $U(1)$ charge k of Section 4 with the number of "up" spins, so the total magnetic quantum number is $M = k - n/2$. The dimension of the fixed- M subspace is exactly the $U(1)$ sector dimension already derived,

$$\dim(M) = \binom{n}{n/2 + M}. \quad (14)$$

Every $SU(2)$ irreducible representation of spin $j' \geq |M|$ contributes exactly one basis state to the fixed- M subspace (the state $|j', M\rangle$), so

$$\dim(M) = \sum_{j' \geq M, j' - M \in \mathbb{Z}_{\geq 0}} m(n, j'). \quad (15)$$

Subtracting the $M = j + 1$ instance of Eq. (15) from the $M = j$ instance removes every term with $j' \geq j + 1$ from both sides, leaving exactly the single term $m(n, j)$ on the right:

$$m(n, j) = \dim(j) - \dim(j+1) = \binom{n}{n/2 + j} - \binom{n}{n/2 + j + 1}. \quad (16)$$

Eq. (16) is identical to Eq. (9) after applying $\binom{n}{k} = \binom{n}{n-k}$ to both binomials ($n - (n/2 + j) = n/2 - j$ and $n - (n/2 + j + 1) = n/2 - j - 1$), giving a second, independent route to the same closed form — one via the branching recursion (Section 4), one via telescoping the same $U(1)$ counts used for the qubit's sector argument in Section 5. Agreement between the two is itself a check on both.

Numerical cross-check. `check_qubit_compass_recovery.py` builds the spin content of n spin- $\frac{1}{2}$'s by direct, repeated Clebsch-Gordan branching (Eq. (8), computed with no reference to the closed form) for $n = 1$ through 10, and compares every resulting multiplicity against Eq. (9) evaluated independently. All entries agree exactly (integer equality, not a numerical tolerance), and the resolution of identity $\sum_j (2j + 1)m(n, j) = 2^n$ holds in every case. The three hand-checked cases quoted in Section 4 ($n = 2, 3, 4$) are reproduced verbatim by this independent computation:

n	$(2j, \text{mult})$ pairs	total dim
2	(0, 1), (2, 1)	4
3	(1, 2), (3, 1)	8
4	(0, 2), (2, 3), (4, 1)	16

No case among $n = 1, \dots, 10$ produced a mismatch between the recursive computation and the closed form, and none produced a resolution-of-identity total different from 2^n .

B Ledger of Every Exact and Structural Check Performed

The scaffold program `check_qubit_compass_recovery.py` performs no symmetric random-circuit simulation, no optimized decoder, and no optimized covariant frame estimator. In its default `-self-check` mode it performs the following checks, each described in the sections above:

1. exact confirmation that the $U(1)$ charge-sector dimensions $\binom{n}{k}$ sum to 2^n for $n = 1, \dots, 12$ (Eq. (7)), with the $n = 2$ and $n = 4$ sector dimensions quoted in Section 4 reproduced directly;
2. an independent recursive Clebsch-Gordan construction of the $SU(2)$ spin content of n spin- $\frac{1}{2}$'s for $n = 1, \dots, 10$, checked term by term (integer equality) against the closed-form Catalan-triangle formula (Eq. (9)) proved two independent ways in Section 4 and Appendix A, together with the resolution of identity $\sum_j (2j+1)m(n, j) = 2^n$;
3. the exact Fisher-information identity of Proposition 2.1, $F_Q = 4 \text{Var}(\hat{J})/\hbar^2$, checked by a central finite difference of an explicitly constructed three-level state against the closed form, and the g -independence of Proposition 2.2, checked by evaluating the same finite difference at five separate base points g_0 and confirming they agree to eight significant figures;
4. unitarity (to machine precision) and exact commutation with $e^{-ig\hat{J}}$ of an explicitly constructed, charge-sector-block-diagonal Haar-random unitary, for $n = 2, 3, 4$ elementary charge units and several random g and random block instances — the finite-size structural object underlying the ansatz of Section 3;
5. exact conservation of the Hilbert-Schmidt norm of an inter-sector coherence block under conjugation by an explicitly constructed block-diagonal isometry (Lemma 6.1), together with exact vanishing of every unrelated output sector-pair block, confirmed on a 3-charge-unit input embedded into arbitrarily larger output sectors;
6. Page's exact average-subsystem-entropy formula (Eq. (11)), evaluated at $m = 2$, $n = 4, 16, 64$, reproducing 0.50952, 0.64652, 0.68144 nats and the residual-gap shrink factors of 3.94 and 3.98 quoted in Section 5;
7. closed-form evaluation, for a solar-mass Schwarzschild hole, of the Schwarzschild radius, β , S/k_B , $\ln S$, t_* (Eq. (10)), and the toy-model crossover tolerance δ^* and the two signed values

of Δt_{LG} at $\delta = 0.1$ and $\delta = 0.5$ rad (Eq. (12)), each checked against an explicit numerical range and, for the two δ values, checked to have the opposite sign the text claims.

The saved output, `scaffold_validation_output.txt`, is the program's real captured standard output from a `-self-check` run; `scientific_result` and `physics_recovery_simulation_run` are both recorded as `false`, and no field reports a recovery time, fidelity, or Fisher-information trajectory as a physical measurement. The program's `-simulate` option exits with status 2 and prints the six-item list of unimplemented prerequisites named in Section 10; it never proceeds to produce a number. A later implementation must not reuse the name `check_qubit_compass_recovery.py` while silently relaxing these seven checks; it must add the optimized-decoder, optimized-estimator, typical-sector-concentration, and charge-sharpening-reconciliation tests named in Section 10, adopt a versioned output schema, and replace `requirements.lock` with an exact, platform-tested dependency lock for whichever tensor-network or exact-diagonalization stack the simulation requires.

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