A Computer Near a Horizon Does Not Get Free Thoughts

Registered research scaffold — no numerical result

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Abstract

A qubit processor held at fixed areal radius near a static black-hole horizon runs on proper time that elapses more slowly than a distant bookkeeper's coordinate time, and this redshift is sometimes read as a subsidy on the number of operations the processor delivers per unit of the distant observer's own resources. This paper registers a candidate resource functional for that claim, the cumulative orthogonalization count $N_\perp[\Gamma]$, built by integrating the tight Margolus–Levitin/Levitin–Toffoli quantum speed limit along a timelike worldline in a fixed background metric, and derives, rather than assumes, the two properties the functional needs before it can be trusted: that the integral is dimensionless because its integrand carries action units, and that a chain of instantaneously bounded orthogonalizations may stand in for a single static bound only inside an explicitly stated quasi-static regime, outside of which the sharper Deffner–Lutz driven-evolution bound governs instead. It then derives, for a static observer in a static spacetime, the identity $E_{\mathrm{loc}}\, d\tau = E_\infty\, dt$ and shows algebraically that pricing $N_\perp[\Gamma]$ from a fixed local energy budget and from a fixed asymptotic Killing budget return the same value, so redshift and blueshift cannot be chained into a net subsidy by a change of accounting convention. The paper then identifies the terms such a claim must still survive: the support acceleration required to hold a static device near a horizon, which diverges there; the blueshifted local Hawking bath; the Landauer cost of erasure; and the Bekenstein capacity ceiling, together with the tidal, causal-duration, and gravitational-decoherence limits on a free-falling alternative and the ordinary energy accounting of the Kerr ergosphere's Penrose process. Two closed-form evaluations anchored to the mass measured for Sagittarius A* and to a stellar-mass merger remnant of the kind reported by LIGO locate where each term dominates; no solver has been implemented, no resource-ledger grid over Schwarzschild and Kerr geometries has been run, and no numerical result beyond these closed-form evaluations is reported. Every claim in this scaffold is either an exact derivation from stated assumptions or a labeled, future-tense research target.

Keywords: quantum speed limit; Margolus–Levitin bound; gravitational redshift; Hawking temperature; Unruh effect; Landauer's principle; Bekenstein bound; Kerr ergosphere; Penrose process.

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Status: Registered scaffold. The driven-evolution regime classifier, the boundary-resolved communication-cost solver, the Schwarzschild/Kerr resource-ledger grid, the priority-search reference floor, and external review are incomplete; Section 10 states each debt and what it blocks.

1 The currency a subsidy must be paid in

A qubit processor held at fixed areal radius r near a Schwarzschild or Kerr horizon ticks through proper time τ more slowly than the coordinate time t used by a distant bookkeeper, by the ordinary redshift factor $\sqrt{f(r)}$. Reversing the ratio produces a tempting inference: if the processor runs at some fixed rate of logical operations per unit of its own proper time, and one proper second down there corresponds to many coordinate seconds up here, then gravity appears to have handed the distant observer computation it did not pay for. This paper's whole object is to state that inference precisely enough to be tested against a resource ledger, and to test it.

The baseline this program inherits, rather than proposes, is the quantum speed limit integrated over ordinary general-relativistic proper time: a bounded-energy system cannot pass between distinguishable states arbitrarily fast, in any spacetime, and nothing about curved geometry is presumed to relax that bound locally. What is at issue is only whether an accounting trick applied across the redshift factor can manufacture an advantage that the local bound, applied honestly, does not permit.

No claim about a computational advantage is well formed until it names a clock and a currency together. Three candidate rates exist and they measure different things: operations per unit of the processor's own proper time, operations per unit of the distant bookkeeper's coordinate time, and operations per unit of energy actually spent, priced in whichever frame that energy was drawn from. The first is a statement about local hardware and has nothing to do with the horizon. The second is a restatement of time dilation and is true by definition. Only the third asks a resource question, because it is the only version in which the redshift factor could in principle appear twice — once as an apparent windfall and once again, unnoticed, inside the very definition of the windfall. Section 5 shows that this double appearance is exactly the error the naive inference makes, and that it cancels rather than compounds once both energy conventions are carried through consistently.

An experimenter who never leaves the distant station has exactly one ledger: whatever energy and time they themselves spend, on their own clock, building, launching, and eventually hearing back from the apparatus held near the horizon. Every other quantity in this problem — proper time at the processor, its local energy gap, the local temperature of whatever bath surrounds it — is a fact about the remote hardware, not a fact about what the experimenter has spent or received. The question this paper answers is whether converting between the remote facts and the experimenter's own ledger, carried out honestly and in full, ever returns a positive balance. Sections 3 through 8 build the object needed to answer it and price every term that could make the answer come out positive; Section 9 and Section 10 state plainly what has been derived, what has only been proposed, and what remains to be computed before any of this is a results paper.

2 Bounds inherited from five literatures

Nothing in this section is a contribution of this paper. It is stated separately from the construction that follows so that no derived or proposed statement in later sections can be mistaken for an inherited one.

Quantum speed limits. A closed system with mean energy $\langle H \rangle$ above its ground-state energy E_0 cannot evolve to an orthogonal state faster than the tight, unified Mandelstam–Tamm/Margolus–Levitin bound established by Levitin and Toffoli [1, 2]. For a Hamiltonian with a time-independent spectrum, Giovannetti, Lloyd, and Maccone showed that this instantaneous rate integrates cleanly over an elapsed time to bound the number of orthogonal states a periodic evolution can pass through [3], and Lloyd applied the same bound to price logical computation directly in terms of available energy [7]. None of these results concerns curved spacetime, and none of them addresses what happens when the Hamiltonian itself is driven in time; the present paper's own contribution begins only where that question does. For a genuinely time-dependent Hamiltonian, Deffner and Lutz derived a geometric bound from the Bures length between instantaneous states that reduces to the naive time-independent integral only in a slowly driven limit and sits below it otherwise [4], building on the geometric approach to quantum evolution of Anandan and Aharonov [6]; Deffner and Campbell's review surveys this family and its regimes of validity [5].

Gravitational time dilation and local energy. That a static observer's proper time relates to coordinate time by $d\tau = \sqrt{f(r)}\,dt$, and that a locally measured quantity is blueshifted relative to its value at infinity by $1/\sqrt{f(r)}$, is ordinary general relativity applied to a static, spherically symmetric metric; the specific extension of this bookkeeping to a locally measured temperature is due to Tolman [14]. Rotating spacetimes replace the static observer with a locally nonrotating (zero-angular-momentum) observer and the lapse function $\sqrt{f}$ with the corresponding Boyer–Lindquist lapse, established by Bardeen, Press, and Teukolsky [17].

Horizon thermodynamics. Hawking's result that a black hole radiates thermally at a temperature fixed by its surface gravity [13] and Unruh's parallel analysis connecting horizon thermality to acceleration [12] are both inherited without modification; Crispino, Higuchi, and Matsas's review collects the accelerated-observer literature this paper draws on for the static-observer support-acceleration argument of Section 6 [15]. Barbado, Barceló, and Garay's analysis of how different observers — static, radially infalling, and otherwise — perceive the same horizon's radiation differently is the specific result this paper leans on when distinguishing the static and free-falling cases in Section 8 [16].

Information-processing floors independent of gravity. Landauer's bound on the heat dissipated by an irreversible bit erasure [8], refined with finite-size corrections by Reeb and Wolf [9], and the Bekenstein bound on the entropy a bounded system of given size and energy can carry [10, 11], are three independently established ceilings that this paper does not modify. Section 7 exists only to keep them from being silently merged with the quantum speed limit under a single word, "information."

Ergosphere energy extraction. That a particle split inside a Kerr ergosphere can return a fragment with more energy than it started with, at the expense of the hole's rotational energy, is Penrose's original mechanism [18]; that this extraction is bounded by the hole's irreducible mass is Christodoulou's result [19], and that a classical horizon's area cannot decrease in any such process is Hawking's area theorem [20]. Together these three results are what license the claim, used without further derivation in Section 8, that Penrose-process energy is drawn from a finite, globally tracked account rather than created.

Anchoring data. The two closed-form evaluations in Sections 6 and 8 use the mass the Event Horizon Telescope Collaboration measured for Sagittarius A* [21] and the progenitor mass scale of the first LIGO/Virgo binary black hole detection [23]; both numbers are used exactly as published, not refit. Pikovski, Zych, Costa, and Brukner's result that gravitational time dilation alone decoheres a composite quantum system carrying internal energy [22] is imported as an additional, independent limit on the free-falling case and is not derived here.

This inventory fixes what the paper is not allowed to reproduce as its own finding. Any displayed statement in Sections 3 onward that merely restates an item from this list without adding a derivation, a dimensional check, or an explicit combination of two or more of these results is a citation error and should be read as one.

3 A dimensionless count along a worldline

Definition 3.1 (Cumulative orthogonalization count)**.** *Let Γ be a timelike worldline in a fixed background spacetime, traced by a quantum processor whose instantaneous mean energy above its own ground state, measured in its own comoving rest frame at proper time τ, is $\langle H(\tau)\rangle - E_0(\tau)$. Define*

$$N_\perp[\Gamma] = \frac{2}{\pi\hbar}\int_\Gamma \big[\langle H(\tau)\rangle - E_0(\tau)\big]\, d\tau. \tag{1}$$

Equation (1) is this paper's proposed object, built entirely from the inherited rate bound of Section 2: inverting $\tau_\perp \geq \pi\hbar/[2(\langle H\rangle - E_0)]$ gives an instantaneous ceiling of $2(\langle H(\tau)\rangle - E_0(\tau))/(\pi\hbar)$ orthogonal state changes per unit proper time, and $N_\perp[\Gamma]$ integrates that ceiling along the worldline exactly as Giovannetti, Lloyd, and Maccone integrate it over ordinary flat-space time [3]. What is new is only the replacement of an unspecified time coordinate by proper time along a worldline in a curved background, together with the two properties proved below, neither of which is automatic once that replacement is made.

Proposition 3.1 (Dimensionlessness)**.** *$N_\perp[\Gamma]$ as defined in Equation (1) is dimensionless.*

Proof. Write out the units of every factor in SI form. The reduced Planck constant carries units of action, $[\hbar] = \mathrm{J\,s}$. The bracketed quantity $\langle H(\tau)\rangle - E_0(\tau)$ is an energy difference, $[\langle H\rangle - E_0] = \mathrm{J}$. The measure $d\tau$ is a proper-time interval, $[d\tau] = \mathrm{s}$. The integral therefore carries units

$$\left[\int_\Gamma (\langle H\rangle - E_0)\, d\tau\right] = \mathrm{J}\cdot\mathrm{s},$$

which is itself an action, the same dimensional class as $\hbar$. Dividing by $\hbar$ leaves

$$\left[\frac{1}{\hbar}\int_\Gamma (\langle H\rangle - E_0)\, d\tau\right] = \frac{\mathrm{J}\cdot\mathrm{s}}{\mathrm{J}\cdot\mathrm{s}} = 1,$$

a pure number, and multiplying by the dimensionless prefactor $2/\pi$ does not change that. Hence $N_\perp[\Gamma]$ is dimensionless. □

This is the check the dossier for this paper demands explicitly, and it is worth stating why it is not vacuous. The integrand of Equation (1) is an energy, not a rate; it is the division by $\hbar$, an action, that converts an action integral into a count. Had the definition instead integrated a *rate* of orthogonalization, $2(\langle H(\tau)\rangle - E_0(\tau))/(\pi\hbar)$, over proper time directly, the result would be exactly the same number by construction, but the two routes make different intermediate quantities dimensionless at different steps, and an implementation that carries units through only one of the two routes is a natural place for an error of exactly one factor of $\hbar$ or one factor of 2π to hide undetected. Appendix A carries both routes through explicitly and confirms they agree.

Proposition 3.2 (Reparametrization and passive–coordinate invariance)**.** *$N_\perp[\Gamma]$ is unchanged under a smooth reparametrization of the curve parameter along Γ and under any passive coordinate change of the ambient spacetime.*

Proof sketch. Only proper time τ, an affine invariant of the worldline fixed by the spacetime metric and Γ's own tangent vector, enters the integral; a curve-parameter relabeling $\tau \mapsto \sigma(\tau)$ leaves $d\tau$ and the physical content of $\langle H(\tau)\rangle - E_0(\tau)$ unchanged as functions of proper time itself. A passive coordinate change of the spacetime coordinates used to describe the metric does not alter τ, which is defined by $d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu/c^2$ along Γ and is a scalar, nor does it alter the comoving-frame energy $\langle H(\tau)\rangle - E_0(\tau)$, which is evaluated in the processor's own rest frame and is likewise a scalar at each event. □

Remark 3.1 (What invariance does *not* include). $N_\perp[\Gamma]$ is *not* invariant under boosting the local tetrad used to evaluate $\langle H(\tau)\rangle$: a different local observer momentarily coincident with the processor but moving relative to it assigns a different local energy to the same field configuration, exactly as in flat-space relativistic Doppler bookkeeping. This is not a defect; it is the reason the local-energy convention must be pinned explicitly to the processor's own comoving rest frame in every displayed equation of this paper, rather than left implicit as flat-space quantum-speed-limit statements typically leave it. Section 5 treats the one relative-energy ambiguity this paper actually needs to resolve — the choice between a fixed local budget and a fixed asymptotic Killing budget for a *static* observer — and shows that ambiguity is harmless because the two conventions agree once applied consistently. It does not resolve the tetrad-boost ambiguity in general, which is outside this paper's scope.

Definition 3.1 and Propositions 3.1–3.2 are the entire content of what this paper calls the ledger's spine. Everything from here to Section 7 is either a condition under which the spine may legitimately be applied (Section 4), a check on which energy convention feeds it (Section 5), or an additional cost term that a claim built from it must survive (Sections 6–8).

4 The condition that licenses chaining

Definition 3.1 integrates an instantaneous rate bound that was proved for a Hamiltonian with a time-*independent* spectrum [3]. A real computation drives the processor's Hamiltonian through a sequence of distinct instantaneous forms, one per logical gate, and nothing in the inherited result of Section 2 guarantees in advance that stitching together many instantaneously applied bounds produces a valid bound on the whole chain. This section states, rather than assumes, the condition under which that stitching is legitimate.

At each proper time τ along Γ, define the *local orthogonalization time*

$$\tau_{\mathrm{ML}}(\tau) = \frac{\pi\hbar}{2\big[\langle H(\tau)\rangle - E_0(\tau)\big]}, \tag{2}$$

the shortest proper time in which the instantaneous Hamiltonian, held fixed, could carry the processor to an orthogonal state. Treating $\langle H(\tau)\rangle - E_0(\tau)$ as though it were constant over one such interval is only sensible if the gap does not change appreciably while that interval elapses. Formally, require the fractional rate of change of the gap to be small compared with the reciprocal of $\tau_{\mathrm{ML}}(\tau)$:

$$\left|\frac{d}{d\tau}\ln\big[\langle H(\tau)\rangle - E_0(\tau)\big]\right| \ll \frac{1}{\tau_{\mathrm{ML}}(\tau)} = \frac{2\big[\langle H(\tau)\rangle - E_0(\tau)\big]}{\pi\hbar} \tag{3}$$

Proposition 4.1 (Units of the chaining condition). *Both sides of Equation (3) carry units of inverse time.*

Proof. The logarithm of a ratio of two energies is dimensionless, so its proper-time derivative on the left carries units s^{-1}. On the right, $[\langle H\rangle - E_0] = \mathrm{J}$ and $[\hbar] = \mathrm{J\,s}$, so $2[\langle H\rangle - E_0]/(\pi\hbar)$ carries units $\mathrm{J}/(\mathrm{J\,s}) = \mathrm{s}^{-1}$. The inequality compares like with like. $\square$

Where Equation (3) holds, the gap is well approximated as frozen for the duration of its own local orthogonalization time, each successive orthogonalization can be assigned to a definite short interval of Γ governed by the instantaneous Margolus–Levitin rate, and summing — equivalently, integrating — those instantaneous rates along Γ reproduces exactly the construction of Definition 3.1. This is the quasi-static regime, and it is the only regime in which this paper's central object is licensed as a rigorous upper bound rather than adopted by analogy.

Where Equation (3) fails — a gate driven on a timescale comparable to or shorter than its own local orthogonalization time — $N_\perp[\Gamma]$ as defined is not known to bound the achievable orthogonalization rate, and the relevant limit is instead the sharper, unconditionally valid geometric bound of Deffner and Lutz, built from the Bures length traced out by the instantaneous state along its actual time-dependent evolution [4], following the geometric formulation of quantum evolution due to Anandan and Aharonov [6]. That bound reduces to the naive integral of Equation (1) precisely in the slowly driven limit and sits at or below it otherwise, so nothing in this paper's spine is contradicted by rapid driving; rather, rapid driving moves the correct bound outside the family this paper analyzes.

Remark 4.1 (Why this matters for a horizon claim specifically). A claim that a horizon subsidizes computation has to survive being applied to a device that is actually running gates, not to an idealized system passively accumulating proper time. Equation (3) is a statement about the processor's own internal driving schedule, not about the metric, and it holds or fails independently of how close the device is to a horizon. It is stated here, ahead of any gravitational content, precisely because it would otherwise be tempting to assume the static-metric identity derived in Section 5 settles the whole chaining question; it settles only the question of which energy convention to use once Equation (3) already holds. Section 10 records that no classification of realistic gate schedules against this condition has yet been carried out for any specific hardware model, and that every subsequent numerical statement in this paper is conditioned on the regime, not asserted unconditionally.

5 One factor and its own reciprocal

Restrict attention now to a static observer at fixed areal radius r in a static, spherically symmetric spacetime, with $d\tau = \sqrt{f(r)}\,dt$ and, for Schwarzschild, $f(r) = 1 - 2GM/(rc^2)$. Two conventions exist for pricing the energy that feeds Equation (1), and a claimed subsidy lives or dies on whether they agree.

Definition 5.1 (Local and Killing energy conventions)**.** *The* local convention *holds fixed the energy $E_{\rm loc}$ actually measured by the static observer at r — for instance, a battery physically present at the processor delivering a stated amount of locally measured energy. The* asymptotic convention *holds fixed the conserved Killing energy E_∞, the energy the same physical resource would be assigned by a static observer at infinity, related to the local value by the standard blueshift relation*

$$E_{\rm loc} = \frac{E_\infty}{\sqrt{f(r)}}. \tag{4}$$

Proposition 5.1 (Redshift–blueshift cancellation)**.** *For a static observer, $E_{\rm loc}\,d\tau = E_\infty\,dt$, and consequently $N_\perp[\Gamma]$ computed from a fixed local budget over the corresponding proper-time interval equals $N_\perp[\Gamma]$ computed from the corresponding fixed asymptotic budget over the corresponding coordinate-time interval.*

Proof. From $d\tau = \sqrt{f}\,dt$, $dt = d\tau/\sqrt{f}$. Multiply Equation (4) by $d\tau$:

$$E_{\rm loc}\,d\tau = \frac{E_\infty}{\sqrt{f}}\,d\tau = E_\infty\left(\frac{d\tau}{\sqrt{f}}\right) = E_\infty\,dt. \tag{5}$$

Now evaluate $N_\perp$ both ways over a finite interval. Holding $E_{\rm loc} = E_0'$ fixed over a proper-time interval $\Delta\tau$,

$$N_\perp = \frac{2}{\pi\hbar}E_0'\,\Delta\tau. \tag{6}$$

Holding $E_\infty = \sqrt{f}\,E_0'$ fixed instead — the asymptotic price of delivering that same local resource down to r — over the corresponding coordinate interval $\Delta t = \Delta\tau/\sqrt{f}$,

$$N_\perp = \frac{2}{\pi\hbar}E_\infty\,\Delta t = \frac{2}{\pi\hbar}\big(\sqrt{f}\,E_0'\big)\frac{\Delta\tau}{\sqrt{f}} = \frac{2}{\pi\hbar}E_0'\,\Delta\tau. \tag{7}$$

Equations (6) and (7) are identical. ☐

This is a covariance check on Definition 3.1, not a new conservation law: it says $N_\perp[\Gamma]$ does not care which energy convention prices it, provided the convention is applied consistently — local energy paired with the proper-time interval it actually spans, or Killing energy paired with the coordinate-time interval that same physical episode spans, never a local energy stapled to a coordinate-time integral or the reverse. Appendix A verifies the same cancellation holds along a graded sequence of static radii rather than only at a single fixed r, and Appendix B records that the check is implemented numerically as well as proved here.

The naive subsidy argument of Section 1 is now nameable precisely: it takes the large blueshift factor $1/\sqrt{f}$ multiplying local energy and separately takes the large factor $1/\sqrt{f}$ multiplying elapsed coordinate time, and treats the two as independent windfalls rather than as one factor and its own reciprocal appearing on opposite sides of Equation (5). An identity cannot fund a subsidy; the same discipline already governs any energy-conversion bookkeeping in general relativity, for instance the exact cancellation of blueshift on the way down and redshift on the way back up for a photon lowered and raised in a static field. Nothing in Proposition 5.1 says a horizon offers no advantage anywhere in the physical situation — it says only that repackaging the energy convention used to price $N_\perp[\Gamma]$ cannot manufacture one. Whether an advantage survives once the cost of actually holding the processor at r is included is the subject of the next two sections.

6 The price of standing still

Proposition 5.1 shows the energy-convention bookkeeping is neutral; it says nothing about what it costs, in the distant experimenter's own currency, to keep a real processor static at radius r at all. A static observer in Schwarzschild spacetime is not in free fall and must be continuously supported against infall, at proper acceleration

$$a(r) = \frac{GM/r^2}{\sqrt{f(r)}}. \tag{8}$$

Proposition 6.1 (Divergence at the horizon)**.** *As $r \to r_s = 2GM/c^2$, $a(r) \to \infty$, with $a(r) \sim \big(GM/r_s^2\big)\sqrt{r_s/(r-r_s)}$ near the horizon.*

Proof. Write $r = r_s + \epsilon$ with $0 < \epsilon \ll r_s$. Then $f(r) = 1 - r_s/r = \epsilon/(r_s+\epsilon) \approx \epsilon/r_s$, so $\sqrt{f(r)} \approx \sqrt{\epsilon/r_s}$, and $GM/r^2 \to GM/r_s^2$ as $\epsilon \to 0$. Substituting into Equation (8), $a(r_s+\epsilon) \approx (GM/r_s^2)\sqrt{r_s/\epsilon} \to \infty$ as $\epsilon \to 0^+$. ☐

Holding station arbitrarily close to a horizon therefore costs arbitrarily large continuous proper acceleration and, correspondingly, arbitrarily large continuously supplied energy — the same divergence that the accelerated-observer literature treats as the flip side of a diverging local temperature for a static or uniformly accelerated observer [12, 15].

Closed-form evaluation. Evaluating Equation (8) at the mass the Event Horizon Telescope Collaboration measured for Sagittarius A*, $M \approx 4 \times 10^6\,M_\odot$

[21], gives a Schwarzschild radius $r_s \approx 1.18 \times 10^{10}\,\mathrm{m}$ and $GM/r_s^2 \approx 3.80 \times 10^6\,\mathrm{m\,s^{-2}}$. At a clearance of $1\,\mathrm{km}$ above the horizon, $f \approx 8.46 \times 10^{-8}$ and $a \approx 1.3 \times 10^{10}\,\mathrm{m\,s^{-2}}$, more than a billion Earth gravities; even at a clearance equal to fifteen percent of the horizon's own radius, a is still of order 10^6 Earth gravities. This is a direct evaluation of the closed-form expression Equation (8) at a published mass, not a simulated result; Appendix B records the exact arithmetic and its independent verification.

Remark 6.1 (Why this is the term to beat)*.* No structure known to this paper's author survives a sustained proper acceleration within many orders of magnitude of $10^{10}\,\mathrm{m\,s^{-2}}$, so the support term is not a fine correction competing with $N_\perp[\Gamma]$ at the margin; it is the term that removes the scenario from consideration well before the horizon itself is approached, for a supermassive hole. Whether a lighter hole changes this ordering, and whether some non-thrust means of station-keeping — a stable, unpowered orbit unique to strong-field geometry, for instance — could avoid the divergence entirely, are open questions this paper does not resolve; Section 10 states this explicitly as an unclosed gap rather than assuming the static case is representative of every way a processor could be positioned near a horizon.

7 Rate, capacity, and dissipation are three separate ceilings

A processor held near a horizon sits in a bath whose local temperature is blueshifted by the same factor that blueshifts local energy. The Hawking temperature measured at infinity, $T_H = \hbar c^3/(8\pi G M k_B)$ [13], is perceived locally at radius r according to the Tolman relation for a static observer in a static field [14], sharpened for the horizon case by the observer-dependent analysis of Barbado, Barceló, and Garay [16]:

$$T_{\mathrm{loc}}(r) = \frac{T_H}{\sqrt{f(r)}}. \tag{9}$$

Closed-form evaluation. For the Sagittarius A* mass used in Section 6, $T_H \approx 1.5 \times 10^{-14}\,\mathrm{K}$. Setting T_{loc} equal to a demanding dilution-refrigerator base temperature of $10\,\mathrm{mK}$ and solving Equation (9) and $f(r) \approx (r - r_s)/r_s$ for the required clearance gives $r - r_s \approx 2.8 \times 10^{-14}\,\mathrm{m}$, a few nuclear radii from the horizon — twenty-odd orders of magnitude closer than the clearance at which Section 6 already found the support-acceleration term unpayable. For a supermassive hole, cooling is a genuine and non-negotiable line item, but it is not the term that binds first.

This numerical ordering is specific to the mass scale evaluated and is not asserted to hold universally; Section 10 notes that no systematic comparison of the support-acceleration and cooling clearances across the full Schwarzschild mass range has been carried out.

The reason to keep cooling, erasure, and capacity conceptually apart, rather than folding all three into $N_\perp[\Gamma]$, is that they bound three different physical quantities with three different sets of units, and the addendum this cohort works under explicitly warns against blurring them under the single label "information."

- $N_\perp[\Gamma]$, Definition 3.1, bounds a *rate*: orthogonal state changes achievable per unit local energy and proper time, for reversible unitary evolution. It costs no heat in principle.
- Landauer's bound [8], sharpened with finite-size corrections by Reeb and Wolf [9], bounds a *dissipation*: erasing one bit at local temperature T_{loc} costs at least $k_B T_{\mathrm{loc}} \ln 2$ of heat, and applies only to the irreversible subset of operations, register resets and readouts, not to reversible gates.
- The Bekenstein bound [10, 11] bounds a *capacity*: the entropy a system of effective size R and available energy E can carry at all, $S \le 2\pi k_B R E/(\hbar c)$, with no time variable in it whatsoever.

Proposition 7.1 (Dimensional consistency of the Bekenstein bound)**.** *The quantity $2\pi RE/(\hbar c)$ carries units of $k_B^{-1}\times$entropy, so that $S \le 2\pi k_B RE/(\hbar c)$ is dimensionally consistent.*

Proof. $[R] = \mathrm{m}$, $[E] = \mathrm{J}$, $[\hbar] = \mathrm{J\,s}$, $[c] = \mathrm{m\,s^{-1}}$. Then

$$\left[\frac{RE}{\hbar c}\right] = \frac{\mathrm{m}\cdot\mathrm{J}}{\mathrm{J\,s}\cdot\mathrm{m\,s^{-1}}} = \frac{\mathrm{m\,J}}{\mathrm{J\,m}} = 1,$$

a pure number, so $[2\pi k_B RE/(\hbar c)] = [k_B]$, the units of entropy (equivalently, of S/k_B as a dimensionless count of nats or bits once k_B is factored out). □

For a processor of size $R = 1\,\mathrm{cm}$ holding $E = 1\,\mathrm{J}$ of available energy, this closed-form bound gives $S \lesssim 27\,\mathrm{J\,K^{-1}}$, of order 10^{24} bits of maximum distinguishable entropy; erasing that many bits at $10\,\mathrm{mK}$ would cost, by Landauer's bound, a substantial fraction of the entire assumed energy budget, entirely independent of any horizon. A processor can be starved by any one of these three ceilings independently, and Section 6's closed-form evaluation shows that near a supermassive horizon the support-acceleration term, which belongs to none of the three ceilings listed here, starves it first, by the largest margin of any term in this ledger.

8 Two routes around the support term, both closed

Section 6 closes the static case for a supermassive hole by ordinary engineering infeasibility. Two candidate routes remain that avoid paying the support-acceleration bill at all, and each is a known limit this construction must reproduce or a countermodel it must survive.

8.1 Free fall pays no support bill, but two other bills come due

A processor in free fall feels no proper acceleration whatsoever, by the equivalence principle, and $N_\perp[\Gamma]$ evaluated along a free-fall worldline reduces to the ordinary flat-space Levitin–Toffoli rate with no support-energy penalty. This is the known limit the dossier for this program states directly: free fall is locally ordinary. Two costs remain even so.

Tidal disruption. A rigid processor of size ℓ falling radially experiences a stretching tidal acceleration between its ends of approximately

$$a_{\mathrm{tidal}}(r) \approx \frac{2GM}{r^3}\,\ell, \tag{10}$$

which at the horizon itself scales as

$$a_{\mathrm{tidal}}(r_s) \approx \frac{\ell\, c^6}{4G^2M^2}, \tag{11}$$

falling as the inverse square of the hole's mass; Appendix A derives Equation (11) from Equation (10) by direct substitution of $r = r_s = 2GM/c^2$. For a laboratory-scale processor with $\ell = 1\,\mathrm{cm}$ at the horizon of a hole with the mass scale of the progenitors of the first LIGO detection, around $30\,M_\odot$ [23], Equation (11) gives $a_{\mathrm{tidal}} \approx 1.1 \times 10^5\,\mathrm{m\,s^{-2}}$, of order ten thousand Earth gravities across one centimeter, sufficient to disassemble ordinary hardware before the horizon is reached. For the Sagittarius A* mass used above, the same formula gives $a_{\mathrm{tidal}} \approx 6 \times 10^{-6}\,\mathrm{m\,s^{-2}}$, negligible. "Free fall is locally ordinary" is therefore not a mass-independent statement; it is bounded, by real astrophysical data, to holes massive enough that Equation (11) stays small over the processor's own size, a condition satisfied at Sagittarius A*'s horizon and badly violated at the stellar-mass horizon LIGO has actually observed.

Causal duration. A free-falling processor that crosses the horizon cannot subsequently transmit its results outward on any reasonable reading of the causal structure; reporting out requires either escaping to infinity, which free fall by definition does not do, or remaining outside, which reintroduces the support-acceleration bill of Section 6. A free-falling processor computes cheaply for exactly as long as it can still deliver the answer, and the ledger this paper proposes must therefore also price the communication step, which Section 10 records as not yet implemented.

Gravitational decoherence at second order. Independent of tidal stress, Pikovski, Zych, Costa, and Brukner show that ordinary gravitational time dilation decoheres a composite quantum system carrying internal energy, because a spatial superposition of such a system becomes correlated with which branch has accumulated more proper time [22]. Near a horizon, where $f(r)$ varies far more sharply with height than in a laboratory, this gradient is set by the same $\sqrt{f}$ that blueshifts everything else in this ledger, and it applies to a free-falling register of finite extent exactly as it applies to a static one. "Free fall is locally ordinary" is a first-order statement, valid only to the extent that this second-order effect stays small across the register's own size and coherence time; this paper imports the mechanism rather than deriving its horizon-specific magnitude, and Section 10 records that magnitude as unevaluated.

8.2 The ergosphere offers credit, not a gift

Inside a Kerr ergosphere no observer can remain static relative to infinity; a locally nonrotating observer is instead dragged around the hole at a rate fixed by the metric, with a redshift-like relation generalizing Equation (4) to the Boyer–Lindquist lapse function [17]. The ergosphere is also where the Penrose process operates: a particle split there can send one fragment outward carrying more energy than the original particle possessed, the deficit paid by the hole's rotational energy [18]. This raises directly the question of whether ergosphere energy extraction is the loophole the static case could not supply — energy generated inside the ergosphere rather than delivered there from outside, available to power computation without Section 6's delivery cost.

It is not, for two reasons internal to this paper's own ledger rather than to new physics. First, the energy extracted is drawn from a real, finite, globally tracked reservoir: the hole's irreducible mass sets a hard ceiling on the rotational energy any sequence of Penrose processes can remove [19], and the horizon area cannot decrease in the process [20], so "free" energy here means energy billed to the hole's own account, not energy created from nothing. Second, and more directly relevant to $N_\perp[\Gamma]$: getting apparatus into the ergosphere to perform the split, and getting the energetic fragment back out to where a distant experimenter can use it, is subject to the same class of redshift/blueshift bookkeeping proved in Section 5, now expressed through the ZAMO lapse function rather than $\sqrt{f}$. The Penrose process changes which reservoir pays — the hole's spin rather than an external battery — but does not change the fact that whatever reaches the distant experimenter's ledger as usable energy is exactly what was extracted, priced in the experimenter's own currency, with no residual left over to fund an orthogonalization count beyond what that energy licenses under Equation (1).

Remark 8.1 (What would reopen this). The one gap this section leaves open rather than closes by calculation is a genuine mechanism for holding station near a horizon that requires no continuously supplied proper acceleration — some regime of stable, unpowered orbital support unique to strong-field Kerr geometry, evaluated with the rigor Section 6 applies to the Schwarzschild static case. This paper constructs no such mechanism and evaluates none; Section 10 states this as the single largest unclosed line item in an otherwise closed ledger, rather than treating its absence as evidence that no such mechanism exists.

9 How this ledger could be wrong

The kill criterion registered for this program is arithmetic rather than interpretive: any claimed computational advantage must vanish once support energy, cooling energy, communication energy, and a consistently applied energy convention are all included in the same ledger. Sections 5–8 satisfy that criterion for the specific static-Schwarzschild, free-fall, and Kerr-ergosphere cases actually evaluated. The claim is falsifiable, and this section states what would falsify it, sorted from the failure that would most damage the construction to the least.

A working exception to Section 6. If a genuine, calculable regime of stable, unpowered station-keeping near a Kerr horizon exists — and Section 8 explicitly leaves this open rather than ruling it out — then the support-acceleration term, the largest line item in this entire ledger, would need to be replaced by whatever finite cost that regime actually carries, and the ordering established in Section 7 between the support and cooling terms could change entirely for the mass range where such orbits exist.

Failure of the quasi-static condition for realistic hardware. If Equation (3) fails for gate schedules any real processor would actually run, then Definition 3.1 is the wrong bound to be chaining at all, and every downstream evaluation in this paper inherits that error; the correct comparison would have to be rebuilt from the Deffner–Lutz driven bound [4] along the same worldlines, which this paper has not attempted.

An error in the redshift/blueshift identity as applied here. Proposition 5.1 is elementary algebra given $d\tau = \sqrt{f}\,dt$ and Equation (4), but an error in how either energy convention is defined operationally — for instance, silently mixing a fixed local budget with a coordinate-time integral, the exact error this paper accuses the naive subsidy argument of making — would reintroduce the apparent subsidy through the same route this paper closes. Appendix B records the analytic and numerical checks run against exactly this failure mode.

Mass-scale dependence not yet mapped. Both closed-form evaluations in this paper use one supermassive mass and one stellar mass; no continuous scan across the Schwarzschild mass range has been performed to confirm that the support term dominates the cooling term everywhere, or to locate the mass, if one exists, at which the ordering of Section 7 reverses. A future calculation finding such a crossover at an astrophysically relevant mass would qualify, not overturn, this paper's conclusion.

Gravitational decoherence magnitude. The gravitational time-dilation decoherence mechanism of Pikovski, Zych, Costa, and Brukner [22] is imported in Section 8 as a qualitative limit on the free-fall case; its magnitude for a specific register size and coherence time near a specific horizon has not been evaluated, and if that magnitude turns out to be negligible over realistic register scales, the free-fall case would be limited by tidal stress and causal duration alone, unchanged in its conclusion but simplified in its argument.

Priority. The reference set assembled for this scaffold is deliberately narrow and has not been checked against a full field-wide priority search of the kind the cohort's freeze gate ultimately requires; if a comparable resource-accounting ledger for near-horizon computation already exists in the literature under different notation, this paper's contribution narrows to whatever combination of the chaining condition, the identity proof, and the specific closed-form evaluations is not already present there.

None of these failure modes has been triggered by anything computed in this paper, because nothing beyond closed-form evaluation of already-known expressions at fixed, cited masses has been computed. They are recorded here as the conditions under which the conclusion would have to be revised, not as evidence against it.

10 Five entries this ledger cannot yet price

Every derivation in Sections 3–8 is exact given its stated assumptions, and both closed-form numerical evaluations in Sections 6–8 are direct substitutions of published masses into closed-form expressions, independently checked in Appendix B. None of that constitutes the resource-ledger grid the research program actually requires. This section states each remaining debt plainly, in the order it must be paid, so that no reader mistakes a derivation for a result and no future draft of this paper can quietly skip a step by rewriting the surrounding prose.

Debt 1 — the quasi-static classifier. Section 4 states Equation (3) as the condition licensing Definition 3.1, but no gate-schedule model for any real or proposed processor architecture has been checked against it. Until at least one concrete driving schedule is classified as quasi-static or not, every numerical use of $N_\perp[\Gamma]$ in this paper — including the two closed-form evaluations already presented — should be read as conditional on an assumption that has not yet been tested against hardware, not as a statement about any specific device.

Debt 2 — the resource-ledger grid. The dossier for this program calls for a resource ledger comparing static and free-falling processors in Schwarzschild and Kerr spacetimes, under both the local and asymptotic energy conventions, across a range of masses, radii, and spins. No such grid has been computed. The two closed-form evaluations in this paper are single points, chosen because they are anchored to measured masses, not a scan; no interpolation between them, and no claim about where a crossover between the support and cooling terms occurs at intermediate mass, should be inferred from this paper until that grid exists.

Debt 3 — the communication-cost solver. Section 8 identifies the cost of reporting a result outward, whether from a static device via ordinary signaling or from a free-falling one before it crosses the horizon, as a required ledger line item, and prices none of it. Building this term requires a boundary-resolved model of signal propagation in the relevant metric that this scaffold does not attempt.

Debt 4 — the gravitational-decoherence magnitude. The mechanism of Pikovski, Zych, Costa, and Brukner [22] is cited qualitatively in Section 8 as a limit on the free-fall case. Its magnitude for any specific register size, internal energy spread, and horizon mass has not been calculated, and the free-fall conclusion in this paper does not currently depend on that magnitude being large; it should not be read as though it does until the calculation exists.

Debt 5 — priority and external review. The reference set in this paper's bibliography was assembled to support the specific derivations made here, not from a completed field-wide priority search of the quantum-speed-limit, near-horizon-thermodynamics, and Penrose-process literatures jointly; that search, an appointed external subject-matter reviewer's assessment of Section 4's chaining condition and Section 5's identity in particular, and the sixty-plus-source floor this cohort's architecture ultimately requires are all outstanding. No claim of novelty beyond the specific combination of results assembled here should be read into this draft.

Debt 6 — the test ledger's own scope. Appendix B records what `check_orthogonalization_ledger.py` actually verifies today: dimensional consistency, the flat-space recovery of the Margolus–Levitin bound, the redshift/blueshift identity, monotonicity of the support-acceleration divergence, and the two closed-form numerical evaluations already quoted in the main text. It does not implement, and its `-simulate` mode refuses to run, any part of Debts 1–3 above; the refusal message names them explicitly rather than failing silently.

Debts 1 and 2 block every subsequent numerical claim this program could make; Debt 3 is required before the free-fall case can be considered fully priced; Debt 4 is a magnitude check on an already-qualitative limit; Debt 5 is a scholarly precondition rather than a calculation; Debt 6 is downstream bookkeeping. None of the derivations in Sections 3–5 depends on any of these six debts being paid, and none of the six debts, once paid, is expected to overturn those derivations — but all six stand between this scaffold and a paper entitled to report a result.

11 The ledger as it stands

This scaffold registers one object, $N_\perp[\Gamma]$, built entirely from an inherited quantum speed limit integrated along a worldline's own proper time, and proves two properties of that object before allowing it to be used: that it is dimensionless, because its integrand is an energy and its measure a time and their ratio to $\hbar$ a pure number, and that its construction as a chained sum of instantaneous bounds is licensed only in an explicitly stated quasi-static regime, outside of which a different, already-established bound governs instead. It then proves, for a static observer in a static spacetime, that the local and asymptotic energy conventions price this object identically, so that no accounting maneuver across the redshift factor can manufacture a subsidy from the identity alone. Against that settled bookkeeping, it prices the terms an actual subsidy claim would have to survive — diverging support acceleration for a static device, a blueshifted local bath, tidal disruption and causal duration for a free-falling one, and the ordinary energy accounting of ergosphere extraction for a rotating hole — and finds, in the two closed-form evaluations it actually carries out, that the support-acceleration term dominates by the largest margin of anything in the ledger.

It reports no resource-ledger grid, no communication-cost solver, no gravitational-decoherence magnitude, and no completed priority search. Those six debts are listed in Section 10 in the order they must be paid, and paying them is what would convert this scaffold into a paper entitled to report a result. If the ledger survives that work, it will have shown that the quantum speed limit and general relativity's own energy bookkeeping already close a claim that has circulated informally without ever being priced; if some unclosed line item — most plausibly the unpowered-station-keeping gap left open in Section 8 — turns out to reopen the claim for some regime, the inherited theories this paper leans on throughout, the quantum speed limit, gravitational redshift, horizon thermodynamics, Landauer's principle, and the Bekenstein bound, remain exactly as intact as they were before this program began.

The deliverable at this stage is a falsifiable registration of what would have to be true for a horizon to subsidize computation, together with a demonstration that, on every term this paper has actually been able to price in closed form, it is not.

A Dimensional and limiting-case derivations

This appendix carries out, in full, the derivations that Sections 3–8 state or sketch. Nothing here is a numerical result; every item is either an algebraic identity or a substitution of a published constant into a closed-form expression already displayed in the main text.

A.1 Two routes to the same dimensionless count

Definition 3.1 can be reached by integrating an energy over proper time and dividing once by $\hbar$ (the route used in Section 3), or by integrating an already-dimensionless-per-unit-time rate over proper time directly. Route one:

$$N_\perp = \frac{2}{\pi\hbar}\int_\Gamma \left[\langle H(\tau)\rangle - E_0(\tau)\right] d\tau, \qquad \left[\frac{\mathrm{J}\cdot\mathrm{s}}{\mathrm{J}\cdot\mathrm{s}}\right] = 1.$$

Route two defines the instantaneous rate $\nu(\tau) \equiv 2[\langle H(\tau)\rangle - E_0(\tau)]/(\pi\hbar)$, with units $[\nu] = \mathrm{J}/(\mathrm{J\,s}) = \mathrm{s}^{-1}$, and integrates:

$$N_\perp = \int_\Gamma \nu(\tau)\, d\tau, \qquad [\nu\, d\tau] = \mathrm{s}^{-1}\cdot\mathrm{s} = 1.$$

Both routes give the identical integral, since $\nu(\tau)\,d\tau = \frac{2}{\pi\hbar}[\langle H(\tau)\rangle - E_0(\tau)]\,d\tau$ term by term; they are recorded separately here because an implementation error of one factor of $\hbar$ or 2π would leave route one looking dimensionally fine (both numerator and denominator are actions) while breaking route two's per-step unit of s^{-1}, or vice versa. The self-check program described in Appendix B evaluates both routes on the same synthetic worldline and requires them to agree to floating-point precision.

A.2 Flat-spacetime recovery

Set $f \equiv 1$ everywhere, the flat-spacetime case. Then $d\tau = dt$ identically, the processor's comoving frame coincides with the lab frame, and Definition 3.1 becomes

$$N_\perp[\Gamma] = \frac{2}{\pi\hbar}\int_{t_1}^{t_2} \left[\langle H(t)\rangle - E_0(t)\right] dt,$$

which for a time-independent gap $\langle H\rangle - E_0$ reproduces exactly the periodic-evolution bound $N \le (2/\pi\hbar)(\langle H\rangle - E_0)\,t$ of Giovannetti, Lloyd, and Maccone [3] with $t = t_2 - t_1$. No curvature-dependent factor survives this limit, confirming that Definition 3.1 is a strict generalization of the inherited flat-space bound and not an independent postulate that merely resembles it.

A.3 Static-observer proper acceleration

For a static metric of the form $ds^2 = -f(r)c^2\,dt^2 + dr^2/f(r) + r^2 d\Omega^2$, a static observer's four-velocity is $u^\mu = (1/\sqrt{f}, 0, 0, 0)$, normalized so that $g_{\mu\nu}u^\mu u^\nu = -c^2$. Because u^μ has a constant t-component along the observer's worldline, the proper acceleration $a^\mu = u^\nu\nabla_\nu u^\mu$ reduces to $a^\mu = \Gamma^\mu_{tt}(u^t)^2$, and the only nonzero component for this metric form is radial, giving the standard result

$$a(r) = \frac{c^2}{2}\frac{f'(r)}{\sqrt{f(r)}}. \tag{12}$$

For Schwarzschild, $f(r) = 1 - 2GM/(rc^2)$, so $f'(r) = 2GM/(r^2c^2)$, and substituting into Equation (12) gives

$$a(r) = \frac{c^2}{2}\cdot\frac{2GM/(r^2c^2)}{\sqrt{f(r)}} = \frac{GM/r^2}{\sqrt{f(r)}},$$

which is Equation (8). Units: $[c^2] = \mathrm{m}^2\,\mathrm{s}^{-2}$, $[f'] = \mathrm{m}^{-1}$ since f is dimensionless and r carries units of length, so $[c^2 f'/\sqrt{f}] = \mathrm{m\,s}^{-2}$, an acceleration, as required.

A.4 The redshift/blueshift identity holds at every radius, not only one

Section 5 proves Proposition 5.1 at a fixed r. Because r never appears as a free parameter on either side of Equation (5) once E_{loc}, E_∞, $d\tau$, and dt are related through $f(r)$ at that same r, the identity holds pointwise for every r in the domain where the static metric applies, not merely at the single radius used for illustration; there is no radius-dependent correction term that Equation (5) omits. The self-check program verifies this by evaluating both sides on a grid of r/r_s ratios spanning several orders of magnitude and confirming agreement at each grid point independently, rather than at one point only.

A.5 Tidal acceleration at the horizon

Starting from Equation (10), $a_{\mathrm{tidal}}(r) \approx (2GM/r^3)\ell$, substitute $r = r_s = 2GM/c^2$:

$$a_{\mathrm{tidal}}(r_s) = \frac{2GM\ell}{(2GM/c^2)^3} = \frac{2GM\ell\, c^6}{8G^3M^3} = \frac{\ell\, c^6}{4G^2M^2},$$

which is Equation (11). The inverse-square dependence on M follows directly from this substitution: doubling the hole's mass quarters the tidal stress at its own horizon for a fixed processor size ℓ, which is why a supermassive hole is tidally gentle at its horizon while a stellar-mass hole is not.

B Self-check program and test ledger

The program `check_orthogonalization_ledger.py`, distributed alongside this paper, implements no resource-ledger solver, no gate-schedule classifier, and no communication-cost model. Its default and `-self-check` mode runs seven analytic and closed-form checks against the constructions of Sections 3–8 and Appendix A, using standard-library Python only (`math`, `json`, `argparse`):

1. **Dimensionless routes agree.** The two algebraically equivalent ways of building $N_\perp[\Gamma]$ from Appendix A — integrating an energy and dividing once by $\hbar$, or integrating an already-per-time rate — are evaluated independently on the same synthetic constant-gap worldlines and required to agree to floating-point precision.

2. **Flat-space recovers the Giovannetti–Lloyd–Maccone bound.** With $f = 1$, the curved-spacetime definition of $N_\perp[\Gamma]$ is checked against the flat-space periodic-evolution bound term by term.

3. **Redshift/blueshift identity, on a grid.** Equation (5) and the equality of the local- and asymptotic-convention values of $N_\perp[\Gamma]$ (Proposition 5.1) are checked at eight radii spanning $r/r_s - 1$ from 0.1 down to 10^{-8}, not at one illustrative point only.

4. **Support-acceleration monotonicity and divergence.** Equation (8) is evaluated at shrinking clearances above the Sagittarius A* horizon and checked to increase monotonically as the horizon is approached, to diverge according to the $a \sim (GM/r_s^2)\sqrt{r_s/\epsilon}$ scaling derived in Section 6, and to relax to the flat-space Newtonian value GM/r^2 at large r, with the residual at finite r checked against its predicted $r_s/(2r)$ scaling rather than against an unreachable exact zero.

5. **Unit-rescaling invariance.** $f(r)$ and the Bekenstein dimensionless group $RE/(\hbar c)$ are recomputed after rescaling the mass, length, and time unit basis by three unrelated factors and requiring the physical constants G, c, $\hbar$ to transform according to their known SI dimension; both quantities are confirmed invariant. This is the operational, executable form of the dimensional arguments given symbolically in Section 3 and Section 7.

6. **Tidal horizon-scaling substitution.** Equation (11) is checked against direct substitution of $r = r_s$ into Equation (10) for both mass scales used in Section 8.

7. **Quasi-static regime boundary.** On a synthetic linearly drifting gap, the chaining condition of Equation (3) is confirmed to hold by many orders of magnitude for a slow drift rate and to be violated for a fast one, exercising both sides of the inequality on a controlled case rather than only checking their units symbolically.

The same run also reproduces, from the closed-form expressions and the published Sagittarius A* and LIGO-scale masses, every illustrative number quoted in Sections 6–8: the Schwarzschild radius and horizon-scale acceleration factor for Sagittarius A*, the support acceleration at 1 km and at fifteen percent of r_s clearance, the Hawking temperature and the clearance at which the local bath reaches 10 mK, and the tidal acceleration across a 1 cm processor at both mass scales. These are logged under `illustrative_closed_form_evaluations` in the program's output and are labeled there, and here, as direct evaluations of already-displayed formulas rather than as simulation output.

Running `python3 check_orthogonalization_ledger.py -self-check` exits 0 with all seven checks reporting `pass`; the saved output is reproduced verbatim in `scaffold_validation_output.txt` and was not hand-edited. Running the same program with `-simulate` exits 2 and prints, to standard error, the five debts enumerated in Section 10 by name; no simulation runs and no additional output is produced. A later implementation that adds the quasi-static classifier, the resource-ledger grid, or the communication-cost solver must not silently reuse the name `check_orthogonalization_ledger.py` while replacing these seven gates — it must add tests for the new code, write a versioned output schema, and record a dependency lock for whatever curved-spacetime and optimization stack the grid computation ends up requiring, none of which is needed for the analytic checks this version performs.

References

[1] N. Margolus and L. B. Levitin (1998). The maximum speed of dynamical evolution. *Physica D: Nonlinear Phenomena* 120(1–2), 188–195. https://doi.org/10.1016/S0167-2789(98)00054-2.

[2] L. B. Levitin and T. Toffoli (2009). Fundamental limit on the rate of quantum dynamics: the unified bound is tight. *Physical Review Letters* 103, 160502. https://doi.org/10.1103/PhysRevLett.103.160502.

[3] V. Giovannetti, S. Lloyd, and L. Maccone (2003). Quantum limits to dynamical evolution. *Physical Review A* 67, 052109. https://doi.org/10.1103/PhysRevA.67.052109.

[4] S. Deffner and E. Lutz (2013). Energy–time uncertainty relation for driven quantum systems. *Journal of Physics A: Mathematical and Theoretical* 46(33), 335302. https://doi.org/10.1088/1751-8113/46/33/335302.

[5] S. Deffner and S. Campbell (2017). Quantum speed limits: from Heisenberg's uncertainty principle to optimal quantum control. *Journal of Physics A: Mathematical and Theoretical* 50(45), 453001. https://doi.org/10.1088/1751-8121/aa86c6.

[6] J. Anandan and Y. Aharonov (1990). Geometry of quantum evolution. *Physical Review Letters* 65(14), 1697–1700. https://doi.org/10.1103/PhysRevLett.65.1697.

[7] S. Lloyd (2000). Ultimate physical limits to computation. *Nature* 406(6799), 1047–1054. https://doi.org/10.1038/35023282.

[8] R. Landauer (1961). Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development* 5(3), 183–191. https://doi.org/10.1147/rd.53.0183.

[9] D. Reeb and M. M. Wolf (2014). An improved Landauer principle with finite-size corrections. *New Journal of Physics* 16, 103011. https://doi.org/10.1088/1367-2630/16/10/103011.

[10] J. D. Bekenstein (1973). Black holes and entropy. *Physical Review D* 7(8), 2333–2346. https://doi.org/10.1103/PhysRevD.7.2333.

[11] J. D. Bekenstein (1981). Universal upper bound on the entropy-to-energy ratio for bounded systems. *Physical Review D* 23(2), 287–298. https://doi.org/10.1103/PhysRevD.23.287.

[12] W. G. Unruh (1976). Notes on black-hole evaporation. *Physical Review D* 14(4), 870–892. https://doi.org/10.1103/PhysRevD.14.870.

[13] S. W. Hawking (1975). Particle creation by black holes. *Communications in Mathematical Physics* 43(3), 199–220. https://doi.org/10.1007/BF02345020.

[14] R. C. Tolman (1930). On the weight of heat and thermal equilibrium in general relativity. *Physical Review* 35(8), 904–924. https://doi.org/10.1103/PhysRev.35.904.

[15] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas (2008). The Unruh effect and its applications. *Reviews of Modern Physics* 80(3), 787–838. https://doi.org/10.1103/RevModPhys.80.787.

[16] L. C. Barbado, C. Barceló, and L. J. Garay (2011). Hawking radiation as perceived by different observers. *Classical and Quantum Gravity* 28(12), 125021. https://doi.org/10.1088/0264-9381/28/12/125021.

[17] J. M. Bardeen, W. H. Press, and S. A. Teukolsky (1972). Rotating black holes: locally nonrotating frames, energy extraction, and scalar synchrotron radiation. *The Astrophysical Journal* 178, 347. https://doi.org/10.1086/151796.

[18] R. Penrose (1969/2002). "Golden Oldie": gravitational collapse: the role of general relativity. *General Relativity and Gravitation* 34(7), 1141–1165. https://doi.org/10.1023/A:1016578408204.

[19] D. Christodoulou (1970). Reversible and irreversible transformations in black-hole physics. *Physical Review Letters* 25(22), 1596–1597. https://doi.org/10.1103/PhysRevLett.25.1596.

[20] S. W. Hawking (1971). Gravitational radiation from colliding black holes. *Physical Review Letters* 26(21), 1344–1346. https://doi.org/10.1103/PhysRevLett.26.1344.

[21] Event Horizon Telescope Collaboration (2022). First Sagittarius A* Event Horizon Telescope results. I. The shadow of the supermassive black hole in the center of the Milky Way. *The Astrophysical Journal Letters* 930(2), L12. https://doi.org/10.3847/2041-8213/ac6674.

[22] I. Pikovski, M. Zych, F. Costa, and Č. Brukner (2015). Universal decoherence due to gravitational time dilation. *Nature Physics* 11(8), 668–672. https://doi.org/10.1038/nphys3366.

[23] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) (2016). Observation of gravitational waves from a binary black hole merger. *Physical Review Letters* 116(6), 061102. https://doi.org/10.1103/PhysRevLett.116.061102.