Selection Accounting: The Price Equation as the Fundamental Theorem of Evolutionary Economics

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Abstract

Biology's Price equation and empirical economics' plant- and firm-level productivity decompositions are the same accounting identity, discovered twice and never reconciled. The covariance term that splits aggregate productivity growth into reallocation and within-unit improvement is, term for term, Price's split between selection and transmission, and the standard continuer-entrant-exiter decomposition used across the applied literature is that identity applied to one specific partition of the economy. Taking the identification seriously yields three further results. Under replicator share dynamics with a linear selection gradient, the selection component of productivity growth equals selection intensity times the share-weighted variance of log productivity – an economic Fisher theorem with inputs a statistical agency can estimate. Under a neutral null in which share changes carry no information about productivity, the variance of the measured selection term is governed by market concentration: the Herfindahl index is the economy's inverse effective population size, so measured selection in concentrated industries is cheap to manufacture from noise alone, with direct consequences for how the declining-dynamism literature reads its own covariance terms. Iterating the identity over nested levels turns cross-country misallocation measurement into a statement about which level of the economy is selecting and which has its selection suppressed. We read the decomposition literature's own measured magnitudes through this lens, confront the objections the identification invites – firms are not literal replicators, productivity measurement conflates efficiency with market power, the identity cannot establish causation – and close with an evolvability index and five falsifiable predictions against existing longitudinal microdata.

Keywords: Price equation; evolutionary economics; productivity decomposition; reallocation; selection; drift; effective population size; business dynamism.
JEL codes: B52, C73, D24, L11, O47.

1 Introduction

Two literatures spent the second half of the twentieth century solving the same equation without meeting. In 1970, the population geneticist George Price published an exact identity partitioning the change in any population's mean characteristic into a covariance term – selection, the reallocation of frequency toward high-value types – and a transmission term – the realized within-lineage change, weighted by realized fitness [72]. Independently, and for the most part without citing Price at all, empirical industrial organization economists built an entire applied literature around a formally identical decomposition of aggregate productivity growth: Baily, Hulten, and Campbell's plant-level partition [13], Griliches and Regev's share-weighted alternative [42], Olley and Pakes's cross-sectional covariance term [69], and Foster, Haltiwanger, and Krizan's synthesis of the resulting within/between/entry/exit accounting [32], later completed by Melitz and Polanec's dynamic four-term extension [62]. These are not analogous equations. They are the same equation, applied to different populations, discovered twice, and never reconciled term for term. Price's paper sat for years in a biology-only citation network before Frank and others recovered its full generality [34, 36]; the productivity decomposition literature grew up inside labor and industrial organization, built by economists reading Census microdata, none of whom had reason to open a 1970 issue of *Nature*. Disciplinary boundaries, not conceptual difficulty, kept the two apart. This paper does the reconciliation and asks what follows once it is done.

The qualitative half of the answer is not new, and we say so plainly rather than let a reader discover it elsewhere. Metcalfe applied Fisher's principle directly to competing firms under replicator dynamics in the 1990s [63, 64]. Andersen used the Price equation itself, by name, to partition economic evolutionary change into selection and innovation effects [7]. Knudsen formalized the replicator/interactor distinction for firms and routines using the same apparatus [56], building on the research program Nelson and Winter founded around routines as heritable units and market competition as selection on profitability [66]. "Economics is a selection process" is prior art. What is not prior art is what follows once the bridge is worked out term by term against the modern empirical decomposition literature's measured magnitudes, rather than asserted at the level of principle.

The claim ladder

The paper's argument climbs four rungs, each resting on the one below it and each committing to progressively less than a universal law of economic behavior. The point of laying them out explicitly, before any of the mathematics, is that a reader should be able to accept the first rung and reject the fourth without inconsistency: nothing later in the ladder is required for anything earlier in it to be true.

The first rung is an identity (§3, Eq. (1)). Price's covariance equation, applied to firms with shares s_i and log-productivity characteristics z_i, is exact bookkeeping: it holds for any partition of any economy in any period, with no behavioral assumption and no error term. Section 4 shows this identity, applied to the continuer/entrant/exiter partition, *is* the Melitz-Polanec dynamic Olley-Pakes decomposition (Proposition 1) – not an analogue of it, the same algebra.

The second rung is a theorem (§5, Eq. (2)). Under the additional, falsifiable assumption of replicator share dynamics with a linear selection gradient, Proposition 2 states an economic analogue of Fisher's fundamental theorem of natural selection: the selection component of aggregate productivity growth equals selection intensity times the share-weighted variance of log productivity. This is a conditional result, not an identity, and Foster, Haltiwanger, and Syverson's finding that realized selection acts on profitability rather than physical productivity alone means the selection-intensity coefficient is an estimated, market-structure-dependent object, never a constant of nature [33].

The third rung is a null (§8, Eq. (3)). Proposition 4 asks what the selection term looks like when there is, by construction, no selection – when share changes are mean-zero noise uncorrelated with productivity. The answer has a corollary the paper treats as its sharpest new claim: the Herfindahl index is the economy's inverse effective population size, so measured selection in concentrated industries carries drift noise of a specific, computable magnitude, and rising concentration mechanically degrades the statistical credibility of any selection measurement made on that industry.

The fourth rung is a program (§11). If selection, transmission, and drift are all measurable from the same longitudinal microdata, their combination – an evolvability index – is a proposal for what a growth economics that took population thinking seriously would try to estimate next.

Contribution

Positioned against this prior art, the paper's claim to originality is exactly the following, and no more:

1. a systematic, term-by-term bridge between the Price equation and the modern empirical productivity-decomposition literature (Baily-Hulten-Campbell, Griliches-Regev, Olley-Pakes, Foster-Haltiwanger-Krizan, Melitz-Polanec), stated as an exact algebraic correspondence and read against that literature's own measured magnitudes (§§4–6);

2. the drift-null propositions: the neutral-dynamics benchmark for the selection term, and the result that $\text{HHI} = 1/N_e$ identifies the Herfindahl index with the economy's effective population size, yielding an explicit statistical test for selection against drift (§8);

3. a multi-level Price treatment of cross-sectional misallocation measurement, reading Hsieh-Klenow-style allocative losses as suppressed between-unit selection (§7);

4. the evolvability index, proposed as a measurement program with explicit data requirements rather than a finished estimator (§11).

We do not claim to have discovered that economic competition resembles natural selection, that Fisher's principle has an economic reading, or that the Price equation belongs in economics. Those claims belong to Veblen, Alchian, Nelson and Winter, Metcalfe, Andersen, and Knudsen, and are credited to them throughout. What we add is what a synthesis paper is for: taking a qualitative correspondence two literatures already sensed and forcing it through the algebra until it either breaks or yields a numbered proposition with measurable inputs.

Roadmap

Section 2 traces the two lineages historically, from Price's own strange biography through the evolutionary-economics tradition to the empirical decomposition literature that never cited either. Section 3 derives the Price identity in full, states the van Veelen critique that an identity is not a causal theory [93], and fixes the accounting stance this paper takes toward that critique. Section 4 builds the dictionary between the two literatures' vocabularies (Table 1), states Proposition 1, and works a three-firm toy example in closed form. Section 5 states

Proposition 2. Section 6 reviews what the decomposition literature has actually measured – dispersion facts, OP covariance levels across countries, reallocation magnitudes – against Table 4 and Figures 1 and 2. Section 7 extends the identity to nested partitions and states Proposition 3. Section 8 states Proposition 4 and its concentration corollary, with a simulation illustrated in Figure 3. Section 9 treats the transmission term as a feature, not a residual. Section 10 takes the strongest available objections at face value. Section 11 proposes the evolvability index and five falsifiable predictions. Section 12 closes.

2 Two literatures, one identity

The identity at the center of this paper was written down once, by a man with no formal training in evolutionary biology, working without salary at University College London on problems he had picked up only a few years earlier. George Price's route to the covariance equation that bears his name is documented in full in Harman's biography [44]: a chemist by original training who had drifted through industrial research, science journalism, and consulting before arriving in London in 1967 already past fifty and turning, for the first time, to population genetics. Within roughly three years he had derived a single algebraic identity that partitions the change in any character's population average into a covariance term and a transmission term, published it in *Nature* as "Selection and Covariance" [72], and extended the mathematics to weighted averages, general covariances, and nested group structure in a 1972 paper in the *Annals of Human Genetics* [73]. He gave the manuscript to W.D. Hamilton before publication and used it, in the same short burst of productivity, to resolve a long-standing puzzle in the algebra of Fisher's fundamental theorem of natural selection. He then largely left science: an evangelical conversion, an increasingly literal reading of the Good Samaritan parable as a personal obligation, the giving-away of his possessions and income to homeless men he took into his own lodging, and his death by suicide in January 1975 close the biography Harman titles, pointedly, a search for the origins of kindness [44]. The equation outlived the episode by decades and, as this paper argues, has a second, unread life waiting in economics.

What the Price equation became in biology is a small industry of its own. Frank's retrospective identifies three lasting contributions of Price's brief career — the covariance equation itself, the evolutionarily stable strategy concept developed jointly with Maynard Smith [60], and the resolution of Fisher's fundamental theorem — and argues that of the three, the covariance equation is the one whose reach keeps expanding [34]. Frank's later restatements treat it as the exact, assumption-free partition of total evolutionary change into a selection component and a transmission component that underlies kin selection, multilevel selection, and quantitative genetics alike [35, 36]. Queller goes further and argues that the Price equation itself, not Fisher's theorem, is the more deserving bearer of the honorific "fundamental theorem of evolution," since Fisher's theorem, Robertson's secondary theorem, and the breeder's equation can all be recovered from it as special cases [74]. Lehtonen shows that multilevel selection, kin selection, contextual analysis, and the neighbor-modulated approach to social evolution are different readings of the same underlying Price-equation structure rather than competing formalisms [57], and Gardner's primer makes the methodological point this paper leans on most heavily: the equation's value lies not in predicting evolutionary change but in supplying an exact accounting of it [39]. Luque's philosophical treatment places it among the handful of genuine "generalization sketches" in science — structures with the abstractness and invariance to organize disparate domains rather than to model any one of them [58], and Okasha's book-length treatment of the levels-of-selection debate is built substantially on the same apparatus [68]. Grafen's formal Darwinism program runs in parallel, seeking to connect the mathematics of motion in population genetics to the language of fitness-optimization directly [41]. None of this activity, across fifty years, required an economy anywhere in the room.

Economics, meanwhile, had been asking a version of the same question since before Price was born. Veblen's 1898 manifesto demanded to know why economics was not an evolutionary science, indicting the discipline's static, taxonomic habits of mind and calling for an account of economic life as a process of cumulative causation rather than a search for equilibrium states [95]. The demand went unanswered in any formal sense for half a century. Alchian's 1950 paper on uncertainty and evolution supplied the first widely read mechanism: firms need not calculate or maximize anything for competitive markets to behave as if they had, because realized profitability, not foresight, determines which firms survive to be imitated and which are eliminated [5]. Winter's 1964 paper, written independently and close in time, built an explicit formal model of the same claim under the heading of economic "natural selection" [97], and Friedman's methodological essay a few years earlier had already leaned on an informal version of the same selection argument to defend the profit-maximization assumption against the objection that firms plainly do not maximize anything in a calculating sense — natural selection among market participants, Friedman argued, makes the as-if assumption predictively adequate whatever its psychological realism [37]. Penrose's 1952 rejoinder in the *American Economic Review* is the discipline's founding skeptical counterweight: biological analogies applied to firms, she argued, strip out exactly what distinguishes a firm from an organism — conscious, purposive managerial choice under conditions the manager can partly control — and a selection story that erases intentionality is not a theory of the firm at all [71]. That exchange, Alchian and Friedman against Penrose, is the founding controversy this paper inherits and does not resolve; Section 10 returns to it directly.

Nelson and Winter turned the Alchian–Winter selection argument into a research program rather than a debating position. *An Evolutionary Theory of Economic Change* treats organizational routines as the heritable unit, search and innovation as the source of variation, and market competition on realized profitability as the selection mechanism proper, with Schumpeter's account of creative destruction as its acknowledged intellectual ancestor [66, 83]. It is against this program's mathematics, not against Alchian's or Friedman's prose arguments, that the present paper's contribution should be measured, and here the qualitative bridge to Price is already built. Metcalfe's program in the 1990s imported Fisher's principle directly: under replicator share dynamics, the rate of change of an industry's mean productivity is driven by the variance in relative fitness — market-share growth — across its constituent firms, and Metcalfe develops this explicitly, including a chapter titled "Fisher's Principle and the Process of Competition" [63, 64]. This is the direct economics-side precedent for treating a biological fundamental theorem as literally, not metaphorically, applicable to an economy, and Section 5 states its modern, measurable form as Proposition 2 with Metcalfe's priority stated plainly.

The claim that Darwinian principles are ontologically general — that variation, heredity, and selection describe any population of interacting, differentially persisting entities regardless of biological or economic substrate — has its own defended literature, running from Hodgson's argument that the analogy is better read as an ontology than a metaphor [45] through the multi-author defense of "generalized Darwinism" [6] to Hodgson and Knudsen's book-length case that Darwin's own principles, stripped of their genetic particulars, are the correct general theory of any evolutionary process, economic ones included [46]. Surveys by Dosi and Nelson

[28], Witt [98], and Safarzynska and van den Bergh [77] catalog how varied the resulting formal toolkit became — replicator dynamics, evolutionary game theory, agent-based simulation — without converging on a single accounting identity comparable to Price's.

That convergence, in fact, had already been proposed, and honesty about this is owed at the outset. Andersen's 2004 paper is not a distant analogy to the present argument; it is the same argument, made two decades earlier. Under the label of "evometrics," Andersen applies Price's equation directly to populations of firms and industries, partitioning economic evolutionary change into a selection effect and an innovation effect in exactly the covariance-plus-transmission form used throughout this paper [7], with a book-length statistical elaboration of the Schumpeterian setting that motivates it [8]. Knudsen's 2004 paper, independently, builds a general mathematical selection theory that pairs the Price equation with Hull's replicator/interactor distinction and applies the combination to the evolution of organizational routines and habits [56]. Between Metcalfe's Fisher-principle industry dynamics, Andersen's evometrics, and Knudsen's general selection theory, the qualitative identification this paper's title makes — that the Price equation is evolutionary economics' own fundamental theorem — was already on the table by the mid-2000s, and none of what follows claims to have discovered it.

What none of that literature did, and what this paper attempts, is to read backward into an entirely separate body of applied work that had, by the time Andersen and Knudsen were writing, already spent a decade and a half building the empirical apparatus needed to measure Price's two terms directly. Baily, Hulten, and Campbell's plant-level decomposition of manufacturing productivity growth [13], Olley and Pakes's covariance-based decomposition of telecommunications-equipment productivity [69], Griliches and Regev's share-averaging treatment of Israeli industry [42], Foster, Haltiwanger, and Krizan's synthesis and extension across methods [32], and Melitz and Polanec's fully dynamic four-term decomposition with entry and exit [62] were built by national-accounts economists answering to census microdata and growth-accounting conventions, not to population One more strand belongs in this map, because it shows how close mainstream economics came without arriving. Modern industrial organization and growth theory built selection *models* of considerable sophistication — Jovanovic's noisy learning and exit, Hopenhayn's stationary entry–exit equilibrium [48], Ericson and Pakes's Markov-perfect industry dynamics [29], Klepper's life-cycle account of entry timing and shakeout [55], Luttmer's selection-and-imitation theory of the firm-size distribution [59], and the Schumpeterian growth models in which creative destruction is the engine of aggregate productivity [2, 3]. A parallel evolutionary-modelling tradition did the same with explicitly Darwinian machinery: Dosi's technological paradigms and trajectories [27], the Silverberg–Dosi–Orsenigo self-organization model of diffusion [84], Saviotti and Pyka's variety-growth simulations [81], and Arthur's increasing-returns lock-in [10]. What neither strand did is connect its selection dynamics to the exact accounting identity under which the empirical decomposition papers were already, unknowingly, publishing measurements.

genetics, and none of their authors cites Price, Fisher, or the selection literature at all. The covariance term that Olley and Pakes call reallocation and the term Andersen calls selection are, this paper argues, the identical algebraic object computed on the identical kind of data. That identification — term by term, with the productivity-decomposition literature's own published magnitudes read as measurements of Price's selection and transmission components — is the gap the remaining sections fill.

3 The Price equation

Consider a population of units indexed $i = 1, \ldots, N_t$, each carrying a share $s_i(t) \geq 0$ and a characteristic $z_i(t)$. No assumption about what the units are, what generates s_i, or what determines z_i is needed for what follows; the derivation is pure algebra on weighted averages. Define the frequency-weighted expectation, variance, and covariance operators $\mathbb{E}_s[\cdot]$, $\mathrm{Var}_s(\cdot)$, $\mathrm{Cov}_s(\cdot,\cdot)$ using the shares $s_i(t)$ as weights, following Price's own convention: the weights need not sum to one over the domain in question. When the domain is the full population at time t, $\sum_i s_i(t) = 1$ holds by construction and $\mathbb{E}_s[\cdot]$ is an ordinary population average; when the domain is a subset — the continuers C, say, once entrants and exiters are set aside — the weights sum to whatever share of total activity C accounts for, and $\mathbb{E}_s[\cdot]$ is understood to carry the implicit renormalization by that total. This bookkeeping convention is what lets a single formula cover populations whose membership changes between t and $t+1$, and it is the technical root of the entrant/exiter extension taken up below.

Define economic fitness as the gross growth factor of a unit's share, $w_i \equiv s_i(t+1)/s_i(t)$, and let $\bar{w} \equiv \mathbb{E}_s[w_i]$. For the continuer set C, write $z_i(t+1) = z_i(t) + \Delta z_i$ for the within-unit change in the characteristic, and define the share-weighted mean at t as $\bar{z}(t) \equiv \mathbb{E}_s[z_i(t)]$. The corresponding mean at $t+1$, computed using shares renormalized within C, is

$$\bar{z}_C(t+1) \;=\; \frac{\sum_{i\in C} s_i(t+1)\, z_i(t+1)}{\sum_{i\in C} s_i(t+1)} \;=\; \frac{\mathbb{E}_s\big[w_i\big(z_i(t)+\Delta z_i\big)\big]}{\mathbb{E}_s[w_i]} \;=\; \frac{\mathbb{E}_s[w_i z_i(t)] + \mathbb{E}_s[w_i \Delta z_i]}{\bar{w}}.$$

Subtracting $\bar{z}(t)$ and using the identity $\mathbb{E}_s[w_i z_i(t)] = \mathrm{Cov}_s(w_i, z_i) + \bar{w}\,\bar{z}(t)$, which holds by the definition of a weighted covariance, gives

$$\Delta\bar{z} \;=\; \underbrace{\frac{\mathrm{Cov}_s(w_i, z_i)}{\bar{w}}}_{\text{Sel}} + \underbrace{\frac{\mathbb{E}_s[w_i\,\Delta z_i]}{\bar{w}}}_{\text{Trans}}. \tag{1}$$

This is Price's covariance equation [72], extended to weighted averages exactly as in his 1972 paper [73] and restated in its modern, textbook form by Frank and by Rice [36, 76]. Read left to right: $\Delta\bar{z}$ is the change in the population's activity-weighted mean characteristic — in the economic application, aggregate log productivity. Sel, the covariance term, is the part of that change attributable to reallocation: units with above-average fitness w_i are, by definition, gaining share, and if fitness covaries positively with z_i then share is moving systematically toward high-z units, raising the mean even if no unit's own z_i ever changes. Trans, the second term, is the fitness-weighted average of within-unit change — the part of $\Delta\bar{z}$ that would remain if every unit kept its time-t share exactly. Every term on the right of (1) is computed from observables at t and $t+1$; no equilibrium condition, no optimization, no distributional assumption on w_i or z_i, and no claim about what caused the covariance to take the sign or magnitude it did has entered anywhere. This is the paper's exactness claim, and it is worth stating with the same bluntness Gardner uses for the biological case: the value of (1) is not that it predicts $\Delta\bar{z}$, but that it partitions an observed $\Delta\bar{z}$, without residual, into exactly two terms with a fixed and economically legible interpretation [39]. The identity holds for any partition of the economy into units — plants, firms, industries, countries — which is what makes the recursive, multi-level form used in Proposition 4 available at no extra cost, and Okasha's treatment of the levels-of-selection problem is built on exactly this recursive property [68].

The derivation above was stated for a fixed continuer set C, and real economies do not hold still that conveniently: firms enter with no time-t share to compute w_i from, and firms exit with $s_i(t+1) = 0$, contributing a fitness of zero to a population whose surviving members must still be reweighted. Price's original apparatus already tolerates the exit case, since $w_i = 0$ is an ordinary value for the covariance operator; the entry case needs the more general treatment Kerr and Godfrey-Smith supply, in which the ancestor–descendant map between the population at t and the population at $t+1$ is allowed arbitrary connectivity — units with no ancestor, units with no descendant, units with several of either — rather than the one-parent-one-offspring bookkeeping of the textbook case [52]. Under that generalization, the change in the mean characteristic for the *whole* economy, continuers, entrants, and exiters together, decomposes into the continuer-only selection and transmission terms of (1) plus two further terms carrying the entering and exiting units' shares and their characteristic-gaps from the pre-existing mean: an entry term proportional to the entrants' combined share times the gap between entrant productivity and the pre-entry mean, and an exit term of the same form, signed oppositely, for the units that leave. Intuitively, an economy in which entrants are systematically more productive than the units they displace records a positive entry term even if no continuing firm's productivity moves and even if the continuer-level covariance term is exactly zero — entry and exit are themselves a selection channel, operating at the level of which units exist at all rather than which existing units hold more share. That four-term object is not a new equation; it is (1) applied to a population whose membership is allowed to change, and Proposition 1 below identifies it, term for term, with the dynamic decomposition already in use in the productivity literature.

Price's second major result follows from (1) by a single substitution. Fisher's fundamental theorem of natural selection states that the rate of increase in mean fitness attributable to natural selection equals the genetic variance in fitness at that time [31] — a claim whose scope was disputed for four decades because Fisher's own statement of what counts as "attributable to selection alone" is compressed almost to the point of ambiguity. Setting $z_i = w_i$ in (1) isolates exactly that partial change: the selection component of the change in mean fitness is $\text{Cov}_s(w_i, w_i)/\bar{w} = \text{Var}_s(w)/\bar{w}$, Fisher's variance-in-fitness result, recovered as the special case of Price's identity in which the character under study is fitness itself. Price's 1972 paper is the origin of this reading [73]; Frank's unification of Fisher's theorem with Robertson's secondary theorem, Lande–Arnold causal analysis of selection gradients, and Hamilton's rule under the same covariance apparatus is its fullest modern statement [35], and Queller's case that the Price equation, not Fisher's narrower theorem, deserves the title of evolution's fundamental theorem rests on precisely this derivability [74]. Lehtonen's demonstration that multilevel selection, kin selection, and contextual analysis are readings of the same Price-equation structure extends the same point sideways rather than downward [57]. Section 5 performs the identical substitution on economic replicator dynamics to obtain Proposition 2, an economic statement of Fisher's theorem with $\beta\ \text{Var}_s(z)$ in place of $\text{Var}_s(w)/\bar{w}$ and an estimable selection gradient β in place of an unobserved genetic variance.

None of this licenses treating (1) as an explanation of anything, and the sharpest challenge to that temptation comes from within the biological literature itself. Van Veelen's critique distinguishes two very different things a Price-equation manipulation can be doing: a "top-down" derivation within an explicit probability model of a biological process, where the equation's terms inherit causal content from the model, and a purely statistical decomposition of observed data, where they do not [93]. Conflate the two, van Veelen warns, and an accounting identity gets mistaken for a theory — the covariance term is guaranteed to exist and to sum correctly with the transmission term whether or not selection, in any causal sense, produced it. A companion paper sharpens the same worry with explicit counterexamples, showing that the common claim of an equivalence between group selection and kin selection, when the equivalence is derived by Price-equation manipulation alone, does not survive contact with an actual dynamical model [94]. Applied to this paper's project, the criticism lands with full force: nothing in (1) says that the shares $s_i(t)$ move toward high-z_i firms *because* z_i is productivity rather than, say, incumbency, access to credit, or luck; the covariance term is silent on mechanism by construction, and any economic content attached to it — Alchian's selection-on-profitability story, Metcalfe's replicator dynamics, this paper's own βz_i selection gradient in Proposition 2 — is a substantive hypothesis layered on top of an identity that would hold regardless of whether the hypothesis were true.

The stance this paper takes toward that criticism is not to deny it but to relocate it, and the right precedent is not biological but econometric. Solow's growth-accounting residual is, on its own terms, exactly the same kind of object: an exact arithmetic remainder once measured capital and labor growth are subtracted from output growth, computed the same way whether the remainder reflects technical change, measurement error, markup variation, or something else entirely [85]. No economist treats the Solow residual as a theory of technology, and none of the discipline's confidence in growth accounting rests on believing that it is one; its value is that it fixes, once and for all, what any candidate theory of growth has to explain, and it lets studies using different data and different countries be compared on the same footing because the arithmetic behind the number is identical everywhere. Equation (1) is used throughout this paper in exactly that spirit: as bookkeeping, not as a behavioral model, that fixes what Sel and Trans mean well enough that a selection-on- profitability hypothesis (Proposition 2), a null hypothesis of no selection at all (Proposition 3), and a multi-level hypothesis about where selection is suppressed (Proposition 4) can all be stated, measured, and compared using the same two accounting terms. Van Veelen's caution is, on this reading, not an objection to Selection Accounting but a statement of what Selection Accounting is: an identity, not a theory, whose usefulness lies entirely in disciplining the theories built on top of it. This is also why the paper does not follow Grafen's formal Darwinism program, which seeks to derive fitness-optimization language directly from the equations of motion of population genetics [41]: optimization claims of that kind require exactly the top-down modeling structure van Veelen distinguishes from bookkeeping, and an economy's selection gradient β is, as Section 5 emphasizes following Foster, Haltiwanger, and Syverson, an estimated, market-structure-dependent quantity rather than a parameter a formal-Darwinism-style argument could pin down a priori [33]. The identity in (1) is deliberately the weaker, more defensible claim.

Definition 1. *For a unit i present at both t and $t+1$ with activity share $s_i(t) > 0$, economic fitness is $w_i \equiv s_i(t+1)/s_i(t)$, the gross growth factor of the unit's share of aggregate activity. A unit with $w_i > 1$ gains share and is favored by selection in the sense of (1); a unit with $w_i < 1$ loses share; $w_i = 0$ corresponds to exit. No claim about profit, welfare, or efficiency attaches to w_i beyond this definition.*

A final formal remark locates the identity relative to the dynamic it will be paired with in Section 5. The replicator equation of evolutionary game theory [91] — the standard continuous-time law of motion for population shares, developed at textbook depth by Hofbauer and Sigmund [47], Weibull [96], Sandholm [78], and Nowak [67] — is not an alternative to the Price

Table 1: The biology–economics dictionary for selection accounting.

Biological term	Economic term	Measuring literature
Population	Firm population (industry or economy)	[26, 92]
Individual	Plant or firm	[13, 15]
Fitness (w_i)	Share growth (employment or output share)	[51, 69]
Character (z_i)	Log productivity (log TFP)	[88, 89]
Selection term	Between-plant reallocation	[16, 32, 69]
Transmission term	Within-plant productivity change	[18, 42]
Mutation–migration	Entry–exit	[24, 62]

equation but a special case of the dynamics whose per-period accounting Price performs: Page and Nowak show that the two are interconvertible descriptions of one selection process, the replicator equation arising as the Price equation's deterministic flow when fitness is frequency-dependent and transmission is faithful [70]. The identity is the bookkeeping; the replicator equation is one hypothesis about what fills the books.

4 Selection accounting for economies

Identity 1 is exact and general: it holds for any population that can be assigned an activity share summing to one and a scalar characteristic, and it holds without reference to genes, plants, or intent. Section 3 established that generality in the abstract. This section spends it. We first fix a dictionary translating the biological vocabulary of population, individual, fitness, and character into the vocabulary a labor or industrial economist already uses — firm population, plant, share growth, log productivity — together with the empirical literatures that already measure each term, usually without citing Price. We then write Identity 1 out for a three-firm economy and carry every arithmetic step so the reader can check, rather than take on faith, that selection and transmission sum to the observed change in aggregate productivity. Finally we state the paper's first proposition: the entire empirical decomposition literature — Baily–Hulten–Campbell, Griliches–Regev, Foster–Haltiwanger–Krizan, Olley–Pakes, Melitz–Polanec — is this identity, evaluated at different reference points.

Table 1 fixes the correspondence used throughout the paper. Two choices deserve comment before the table speaks for itself. First, the activity share s_i can be built from employment, gross output, or value added; the applied literature is not unanimous, and each choice rescales the units, though not the sign, of every term below. Second, "fitness" in the biological sense — the ratio of a unit's frequency next period to its frequency this period — is measured continuously in the productivity-decomposition literature as share growth, but it is essentially never named as fitness, which is exactly the terminological gap this paper closes.

The mapping deserves the most care at its two ends. Mutation and migration, in Price's original bookkeeping, are the terms that introduce individuals with no ancestor in the previous period's population; in a firm population these are entrants, and firms that fail to survive to the next period play the symmetric role of individuals removed from the population, that is, exiters. Melitz and Polanec's four-term dynamic decomposition treats entry and exit exactly this way

Table 2: A three-firm economy, one period.

Firm	$s_i(t)$	$s_i(t+1)$	w_i	$z_i(t)$	Δz_i	$z_i(t+1)$
A	0.50	0.55	1.10	0.20	0.05	0.25
B	0.30	0.30	1.00	0.00	0.10	0.10
C	0.20	0.15	0.75	−0.30	0.00	−0.30

[62], and this is no accident: their algebra is Price's algebra applied to a population with in- and out-migration, a case whose general covariance form Price himself supplied [73] and which Kerr and Godfrey-Smith later generalized to arbitrary ancestor–descendant connectivity [52]. At the other end, "selection term" is not a metaphor for reallocation; it is the covariance, computed with the same weights and the same characteristic, that the productivity-decomposition literature has been estimating under different names since Baily, Hulten, and Campbell [13].

Identity 1 in economic clothes

Written for a population of firms with activity shares $s_i(t)$ and log productivities $z_i(t)$, Identity 1 reads

$$\Delta\bar{z} \;=\; \underbrace{\frac{\mathrm{Cov}_s(w_i, z_i)}{\bar{w}}}_{\text{Sel}} + \underbrace{\frac{\mathbb{E}_s[w_i\,\Delta z_i]}{\bar{w}}}_{\text{Trans}},$$

with $w_i = s_i(t+1)/s_i(t)$ the gross share-growth factor and $\Delta z_i = z_i(t+1) - z_i(t)$ the within-firm productivity change. Sel is a covariance across firms, computed once, between how much a firm's share is about to grow and how productive it already is. Trans is a weighted average of within-firm productivity growth, the weights being each firm's realized, end-of-period share $s_i(t+1) = s_i(t)\,w_i$. Nothing on the right-hand side is estimated; both terms are computed directly from data any decomposition study already has in hand. In a closed population — no entry or exit within the period, as in the toy economy below — $\bar{w} = \mathbb{E}_s[w_i] = \sum_i s_i(t+1) = 1$ automatically, because shares sum to one at both dates. Once entrants and exiters are added to the accounting, $\bar{w}$ picks up their contribution and no longer collapses to one, which is exactly why Proposition 1, below, needs a separate reference point for each sub-population.

Consider a three-firm economy — call the firms A, B, and C — with activity shares $s_A(t) = 0.50$, $s_B(t) = 0.30$, $s_C(t) = 0.20$ and log productivities $z_A(t) = 0.20$, $z_B(t) = 0.00$, $z_C(t) = -0.30$. Over one period activity reallocates toward the initially more productive firms and away from the initially least productive one: $s_A(t+1) = 0.55$, $s_B(t+1) = 0.30$, $s_C(t+1) = 0.15$. Each firm also changes its own productivity: A improves by $\Delta z_A = 0.05$, B by $\Delta z_B = 0.10$, and C does not improve at all, $\Delta z_C = 0.00$. The table below collects the resulting fitness values $w_i = s_i(t+1)/s_i(t)$ and end-of-period productivities $z_i(t+1) = z_i(t) + \Delta z_i$.

Share-weighted mean productivity starts at $\bar{z}(t) = 0.5(0.20) + 0.3(0.00) + 0.2(-0.30) = 0.10 + 0.00 - 0.06 = 0.04$ and ends at $\bar{z}(t+1) = 0.55(0.25) + 0.30(0.10) + 0.15(-0.30) = 0.1375 + 0.0300 - 0.0450 = 0.1225$, a change of $\Delta\bar{z} = 0.0825$: eight and a quarter log points, roughly $\exp(0.0825) - 1 \approx 8.6\%$ aggregate productivity growth. The two components add up to that number by construction, but it is worth watching them add up. Mean fitness is $\bar{w} = 0.5(1.10) + 0.3(1.00) + 0.2(0.75) = 0.55 + 0.30 + 0.15 = 1.00$, which holds here because the economy is closed: no firm enters or exits, so $\sum_i s_i(t+1) = 1$ regardless of how shares move within the period. The

Table 3: The empirical decompositions as choices of partition and reference point.

Method	Partition	Reference weights	Price-equation reading
BHC [13]	Continuers	Base-period plant shares	Static analogue of Sel; no dynamic fitness term
GR [42]	Continuers	Average of base- and end-period shares	A share-averaging weighting intermediate between the two FHK rows below
FHK M1 / M2 [32]	Continuers, entrants, exiters	Base-period weights (M1) vs. running, end-period weights (M2)	Two reference points for Trans; M1's residual cross term is the piece of Trans that M2 already absorbs
OP [69]	All plants, single cross-section at t	Unweighted mean plus a covariance level	Stock: $\mathrm{cov}_t(s_i, z_i)$ is the running total of past Sel_t
MP [62]	Continuers, entrants, exiters, bias-corrected	Identity 1 applied recursively, own reference point per sub-population	General case; nests the rows above as special reference-point choices

selection covariance is $\mathbb{E}_s[w_i z_i(t)] = 0.5(1.10 \times 0.20) + 0.3(1.00 \times 0.00) + 0.2(0.75 \times -0.30) = 0.1100 + 0.0000 - 0.0450 = 0.0650$, so $\mathrm{Cov}_s(w_i, z_i) = 0.0650 - (1.00)(0.04) = 0.0250$ and $\mathrm{Sel} = 0.0250/1.00 = 0.0250$. Transmission is $\mathbb{E}_s[w_i \Delta z_i] = 0.5(1.10 \times 0.05) + 0.3(1.00 \times 0.10) + 0.2(0.75 \times 0.00) = 0.0275 + 0.0300 + 0.0000 = 0.0575$, so $\mathrm{Trans} = 0.0575/1.00 = 0.0575$. And indeed $\mathrm{Sel} + \mathrm{Trans} = 0.0250 + 0.0575 = 0.0825 = \Delta\bar{z}$, exactly. Of this period's productivity growth, 30.3% (= 0.0250/0.0825) is attributable to reallocation toward the firm that started out more productive, and 69.7% to firms getting better at what they already did. Firm C, which contributed nothing to transmission, still contributed to selection by shrinking; firm B, which gained no share at all ($w_B = 1$), still contributed a positive 0.0300 to transmission, unweighted by any reallocation. A decomposition that tracked only the unweighted cross-firm average of Δz_i — $(0.05 + 0.10 + 0.00)/3 = 0.05$ — would be wrong on two separate counts: it would miss selection entirely, and its within-firm number would use the wrong weights regardless, an issue the rest of this section returns to.

The bridge to the decomposition literature

Proposition 1 (Decomposition bridge)**.** *The Melitz–Polanec dynamic Olley–Pakes decomposition [62] is Identity 1 applied to the partition of the firm population into continuers, entrants, and exiters, with a specific choice of reference weights governing each sub-population's contribution; the Baily–Hulten–Campbell [13], Griliches–Regev [42], and Foster–Haltiwanger–Krizan [32] decompositions are special cases obtained from different choices of reference weights on essentially the same partition. The Olley–Pakes cross-sectional covariance term* $\mathrm{cov}_t(s_i, z_i)$ *[69], computed once at a single date t, is the time-integral stock of the period-by-period flow-selection terms* $\mathrm{Sel}_t = \mathrm{Cov}_s(w_i, z_i)/\bar{w}$ *accumulated up to t. The proof is given in Appendix B.*

The FHK row is where the controversy actually lives. Foster, Haltiwanger, and Krizan compute the same ten-year, four-digit-industry productivity change two ways and get two different-looking answers: under base-period ("method 1") weights, the within-plant component accounts for about half of average industry productivity growth, the between-plant component is negative but small, the cross term is positive and accounts for roughly a third, and net entry contributes 26 percent; under running ("method 2") weights, the within component rises to 65 percent, the between component to 10 percent, and net entry is 25 percent, with no separate cross term reported at all [32]. Nothing about the underlying economy changed between the two rows of their table; only the reference point used to compute the counterfactual moved from the start of the period to the end. Identity 1 shows why the cross term must vanish under one weighting and not the other: Trans $= \mathbb{E}_s[w_i \Delta z_i]/\bar{w}$ is already weighted by each firm's realized, end-of-period fitness w_i — equivalently, by $s_i(t+1)$, exactly as the worked example above made explicit. A decomposition that instead weights within-firm change by base-period shares $s_i(t)$ leaves a residual, $\sum_i [s_i(t+1) - s_i(t)]\, \Delta z_i$, uncounted; that residual is the FHK cross term. It is not a third economic force alongside selection and transmission. It is the wrong reference point for transmission, misfiled as its own row. Price's accounting, which fixes the reference point at realized fitness rather than leaving it a free parameter of the estimator, makes the term disappear because it was never separate to begin with.

Remark 1 (Flow versus stock). Olley and Pakes decompose telecommunications-equipment industry productivity at each date t as an unweighted plant average plus a cross-sectional covariance between output share and productivity level, $\mathrm{cov}_t(s_i, z_i)$; over the industry's 1974–1987 deregulation, that covariance term rose from 0.01 to 0.32 while the unweighted average productivity term fell from 0.90 to 0.66 [69]. This is not a period-by-period selection term; it is a level, computed from a single cross-section, and it is best read as the accumulated trace of thirteen years of period-by-period reallocation, much as a capital stock is the accumulated trace of a flow of investment. Identity 1's Sel_t, by contrast, is a flow: it compares two adjacent periods and answers how much reallocation raised mean productivity in that year alone. A reader watching only the OP stock term climb from 0.01 to 0.32 cannot tell whether reallocation was gradual across the thirteen years or concentrated in the two years immediately after deregulation; a flow decomposition, applied year by year, can. The two objects answer different questions, and neither is more correct than the other. Conflating them — treating a change in the OP stock as though it were a Sel_t flow, or vice versa — is, from a Price-equation standpoint, the single most common misreading of the decomposition literature.

The dictionary and the bridge together license the rest of the paper's strategy. Because the empirical decomposition literature already measures Sel and Trans under half a dozen names and reference conventions, every proposition that follows can be stated with measurable, already-collected inputs rather than newly assumed parameters. Section 5 takes the first of these: what Identity 1 implies once share growth is assumed to respond linearly to productivity.

5 An economic Fisher theorem

Section 4 showed that Identity 1 holds without behavioral content: it is bookkeeping, true for any weights and any characteristic, and true whether or not the units involved are rational, imitative, or even alive. Fisher's fundamental theorem of natural selection is not bookkeeping in that sense — it is a claim about a particular class of dynamics, the ones biologists call replicator dynamics, under which selection alone, holding the underlying character fixed, raises the population mean at a rate equal to the variance the population has to work with. This section states the economic version of that claim, not as a re-derivation of Price's arithmetic but as a genuine restriction: an economy has to look a certain way — its firms' growth rates have to respond linearly to a shared measure of quality — before the closed-form result below applies, and the section spends its second half on how often, and in what sense, that restriction actually holds.

Proposition 2 (An economic Fisher theorem)**.** *Let each firm's activity share evolve under the replicator dynamic*

$$\dot{s}_i = s_i\,(f_i - \bar{f}), \qquad \bar{f} = \mathbb{E}_s[f_i],$$

and let the fitness function be linear in log productivity, $f_i = \beta z_i$*, for a selection intensity* β *common to the population. Then, holding each firm's own productivity* z_i *fixed, the selection component of the growth rate of aggregate log productivity* $Z = \mathbb{E}_s[z_i]$ *is*

$$\dot{Z}\big|_{\text{selection}} \;=\; \mathrm{Cov}_s(f_i, z_i) \;=\; \beta\ \mathrm{Var}_s(z) \;\geq 0. \tag{2}$$

The discrete-time, one-period analogue is $\mathrm{Sel} = \beta\ \mathrm{Var}_s(z)/(1 + \beta\bar{z})$*, which agrees with* (2) *up to an* $O(\Delta z^2)$ *correction arising from the nonlinearity of* $w_i = 1 + f_i$ *and the normalization by* $\bar{w}$*.*

The result is the direct economic translation of Fisher's theorem: the instantaneous contribution of reallocation to productivity growth is not a free parameter but the product of two measurable quantities, a selection intensity and a variance. Every term on the right-hand side of (2) is already collected by the productivity-dispersion literature — $\mathrm{Var}_s(z)$ is exactly the within-industry log-TFP dispersion Syverson measures across U.S. manufacturing plants [88, 89] — so the theorem is not merely elegant; it is falsifiable against numbers that already exist. What it is not is universal: β is a market-specific object, estimated from how strongly a firm's growth actually responds to its productivity rank, and the remainder of this section is about what that estimate is really measuring.

The claim that competition acts on firms the way Fisher's theorem says selection acts on populations is not new to this paper. Metcalfe imported Fisher's principle into industrial economics explicitly, modeling competing firms as a population under replicator dynamics and showing that the rate of change of an industry's mean productivity is governed by the variance in market-share growth across firms [63], a program he later elaborated at book length under the heading of creative destruction [64]. Andersen's Price-equation-based "evometrics" is the same move made with Price's more general bookkeeping rather than Fisher's variance identity specifically [7]. What Proposition 2 adds to that lineage is not the qualitative claim, which is Metcalfe's, but the connection to specific, already-measured empirical objects — dispersion statistics computed by Syverson, Foster, Haltiwanger, and their coauthors from census microdata — that make $\beta\ \mathrm{Var}_s(z)$ a number one can look up an order of magnitude for, rather than a proportionality one can only sign.

Three caveats keep (2) from being read as a law of economic nature.

First, and most consequential: realized selection acts on profitability, not on physical productivity, and the two are not the same variable. Foster, Haltiwanger, and Syverson show that physical, quantity-based productivity is negatively correlated with a plant's price while revenue-based productivity is positively correlated with price [33]; a plant that raises its price loses physical market share for reasons that have nothing to do with technical efficiency, and a decomposition run on revenue-based z_i will assign to Sel whatever reallocation is really being driven by a demand shock or a markup, not a productivity difference. Estimating β from revenue data — the overwhelming majority of the applied literature, since physical-quantity data are rarely available across multi-product firms — therefore estimates a composite object, a mixture of technical selection and demand- or market-power-driven selection, and (2) should be read as decomposing growth in *revenue* productivity into a selection and a transmission piece unless the analyst can show z_i is physical.

Second, and following directly from the first, β is not a constant to be looked up once. It is the semi-elasticity of a firm's growth rate to its productivity rank, and that semi-elasticity depends on the toughness of competition, the elasticity of substitution between varieties, and the speed at which demand can reallocate toward a more efficient producer — market-structure parameters that differ by industry, by country, and by decade. Baqaee and Farhi's finding that reallocation toward high-markup firms, rather than toward high-efficiency firms, accounts for roughly half of aggregate TFP growth in their sample [14] is exactly the failure mode Proposition 2 warns against: a positive, statistically significant $\hat{\beta}\ \mathrm{Var}_s(z)$ can be measuring the market rewarding market power rather than the market rewarding productivity, and only auxiliary evidence on markups, not the Price decomposition itself, can tell the two apart.

Third, demand-driven and technology-driven selection can point in opposite directions at the level of an individual firm even when they agree on average. A plant can be technically excellent and demand-constrained — Penrose's classic case of a firm unable to grow as fast as its own managerial capabilities would allow [71] — or technically mediocre and demand-favored. Because (2) aggregates across the whole population, such cases wash out in $\mathrm{Var}_s(z)$ without disappearing from the underlying economics; a β estimated on average conceals dispersion in the firm-level productivity–growth relationship that a more disaggregated, industry-by-industry selection-gradient estimate would recover.

None of this requires firms to compute anything resembling a Fisher–Price decomposition, or even to be rational optimizers in the textbook sense: the replicator dynamic assumed above has learning-theoretic microfoundations distinct from optimization. Schlag derives the optimal decision rule for a boundedly rational agent choosing among actions of unknown payoff by imitating a randomly sampled peer, switching proportionally more often the larger the peer's realized payoff advantage, and shows that the resulting population-level law of motion is exactly the replicator equation [82]: firms need not know their own productivity rank, only occasionally copy a more successful rival, for the market-share dynamics behind Proposition 2 to emerge in aggregate. Björnerstedt and Weibull show a complementary result, that such imitation dynamics converge to Nash equilibrium play under standard regularity conditions [17], which reconciles the replicator dynamic with the market-selection tradition in economics that treats survival, not calculation, as the operative discipline. That tradition goes back to Alchian's argument that competitive markets can select for approximately profit-maximizing behavior among firms that never optimized anything, because the ones that did not come close enough simply exit [5]. Imitation-based replicator dynamics supply that argument's missing law of motion: not merely that non-maximizers are removed, but a specific functional form, $\dot{s}_i = s_i(f_i - \bar{f})$, for how quickly they are removed — which is what lets (2) be quantitative rather than merely directional. The same survival logic has been pushed all the way into general equilibrium for beliefs rather than technologies: Sandroni and Blume–Easley show that complete markets select for traders whose forecasts are closest to the truth — and that with incomplete markets even that guarantee fails [19, 79] — a reminder that what the market's replicator dynamic rewards depends on the institutions it runs inside, a theme Section 7 returns to.

Taken together, the proposition and its caveats set the empirical agenda for the rest of the paper rather than closing it. (2) says that selection's contribution to productivity growth is a product of two numbers that the microdata literature already reports separately; the caveats say that neither number should be taken at face value without asking what kind of selection — on efficiency or on markup, on physical output or on revenue — is doing the work. Section 6 assembles the magnitudes β and $\mathrm{Var}_s(z)$ actually take across industries and countries, and Section 7 extends the same replicator logic to nested populations, where a positive Sel computed at the plant level can coexist with, or even be produced by, a suppressed Sel one level up.

6 What the measurements say

The identity in (1) is exact by construction, and Propositions 1, 2, and 4 are exact consequences of stated dynamical assumptions; none of this arithmetic guarantees that selection does any real work in an actual economy. That is an empirical question with a definite quantitative answer, and the plant- and firm-level microdata literature built since the early 1990s has answered it in four connected parts: the cross-sectional variance that selection would have to act on is large and persistent; the covariance term that measures selection acting is large, moves with policy, and differs enormously by country; the rate at which the economy manufactures fresh variation through entry has been falling for three decades; and at least one transmissible trait—management practice—shows the persistence and productivity association that a selection account requires of anything it calls a heritable characteristic. Each claim below is tied to the study and figure that established it.

6.1 Dispersion: the raw material of selection

Selection has nothing to act on without variance, and the productivity-dispersion literature's first finding is that there is an enormous amount of it, everywhere, for a long time. Within four-digit U.S. manufacturing industries, the average gap in log total factor productivity between the plant at the 90th percentile and the plant at the 10th percentile is 0.651 log points, a productivity ratio of $e^{0.651} \approx 1.92$: a plant sitting at the top decile of its own industry produces nearly twice the output of a plant at the bottom decile from the same measured capital and labor [88, 89]. This is not a US-specific or narrowly-defined-industry artifact. In Chinese and Indian manufacturing the same 90/10 TFP ratio exceeds 5:1 [49, 89], an order of magnitude more dispersion sitting inside industries nominally producing the same good with the same technology menu. Nor is this dispersion transient noise that would wash out on inspection: regressing a producer's current TFP on its own one-year lag yields autoregressive coefficients on the order of 0.6 to 0.8 [90], so a plant's relative productivity rank is close to a persistent characteristic rather than a draw that resets each period. This is exactly the combination the economic Fisher theorem of §5 needs as an input: $\mathrm{Var}_s(z)$ large, and large in a way that survives period to period rather than averaging itself away, is the raw material that (2) multiplies by selection intensity to produce a nonzero selection contribution to aggregate productivity growth. A world in which within-industry dispersion is a few percentage points and dissipates within a year is a world in which selection accounting is a mathematical curiosity; the world documented by Syverson, Hsieh, and Klenow is not that world.

6.2 Selection at work

The Olley–Pakes covariance term is the empirical decomposition literature's direct measure of Sel: it is the share-weighted covariance between a plant's activity share and its productivity, and under Proposition 1 it is the time-integral stock of past selection acting on the continuer/entrant/exiter partition. Bartelsman, Haltiwanger, and Scarpetta's cross-country comparison gives this quantity its most striking reading. For labor productivity, the OP covariance term averages about 50 log points within U.S. manufacturing industries—in an accounting sense, the average U.S. manufacturing industry's labor-productivity index is 50 percent higher than it would be under random allocation of employment shares. The same term reaches only 20 to 30 log points in Western Europe, and was close to zero, if not negative, in Central and Eastern European economies at the start of their transition away from central planning [16]. Table 5 lays the comparison out. This is not a difference in how productive the average plant in each economy is; it is a difference in whether activity is allocated toward the productive plants once they exist, and it maps directly onto the between-unit selection term this paper's identity isolates.

Selection also visibly responds to policy shocks rather than sitting at some fixed structural level. Olley and Pakes's own telecommunications-equipment study found the reallocation covariance term rising from 0.01 in 1974 to 0.32 in 1987, over the same years that unweighted mean plant productivity in the industry *fell*, from 0.90 to 0.66—industry productivity held roughly flat only because activity shifted decisively toward the plants that remained productive [69]. That thirty-fold rise in the covariance term brackets the AT&T divestiture and the deregulation of telecommunications equipment procurement: selection, in the Price-equation sense, was released by a change in market structure, not by any change in the underlying technology available to plants.

Foster, Haltiwanger, and Krizan's synthesis of the reallocation-decomposition methods gives the canonical ten-year U.S. manufacturing apportionment: net entry accounts for 25 to 26 percent of average industry multifactor productivity growth, and within-plant improvement accounts for 50 to 65 percent depending on decomposition method, with the remainder attributable to reallocation among surviving plants [32]. Net entry's quarter share is itself a selection-on-the-extensive-margin number: it is what happens when low-z incumbents exit and are replaced by higher-z entrants, the between-unit term evaluated across the exiter/entrant boundary rather than within the continuing population. Getting that boundary right is not bookkeeping trivia—Melitz and Polanec show that measuring it with a dynamically consistent decomposition rather than the older static Olley–Pakes method changes the measured contribution of entry and exit to aggregate Slovenian manufacturing productivity growth by up to 10 percentage points [62], which is the empirical content behind Proposition 1's claim that the choice of reference point in the decomposition is a substantive modeling decision, not a normalization.

At the macroeconomic end of the same literature, Baqaee and Farhi's nonparametric general-equilibrium decomposition finds that reallocation of market share toward high-markup firms accounts for about half of aggregate U.S. TFP growth between 1997 and 2015, and that eliminating markup dispersion entirely—pushing every firm's price-cost margin to the same level—would raise aggregate TFP by approximately 15 percent [14]. Read against (1), this is a statement that roughly half of two decades of U.S. productivity growth is the selection term, not the transmission term, and that a substantial share of the remaining productivity is being

Table 4: Headline magnitudes underlying the selection-accounting program. AR(1): first-order autoregressive coefficient of TFP on its own lag. MFP: multifactor productivity.

Fact	Magnitude	Source
US within-4-digit-industry 90/10 TFP ratio	1.92 (0.651 log pts)	[88, 89]
China / India 90/10 TFP ratio	$> 5{:}1$	[49, 89]
Plant/firm TFP AR(1) persistence	0.6–0.8	[90]
US manufacturing OP covariance (labor productivity)	≈50 log pts	[16]
FHK net-entry share of 10-yr manufacturing MFP growth	25–26%	[32]
FHK within-plant share of 10-yr manufacturing MFP growth	50–65%	[32]
Static-vs-dynamic OP decomposition bias (entry/exit)	up to 10 pp	[62]
Reallocation share of US TFP growth, 1997–2015	≈50%	[14]
TFP gain from eliminating markup dispersion	≈15%	[14]
US startup rate: late 1980s → pre-recession → trough	12.0% → 10.6% → <8%	[24]
Management-score effect on productivity (per SD)	3.8–8.0 log pts	[18]

left on the table by an allocation that has not fully tracked underlying z.

6.3 The engine slowing

Selection can only work as fast as the economy replaces its population, and the U.S. rate of replacement has been falling for three decades. Decker, Haltiwanger, Jarmin, and Miranda's calculations from the Business Dynamics Statistics data show the annual startup rate declining from an average of 12.0 percent in the late 1980s to an average of 10.6 percent in the years just before the Great Recession, and then falling below 8 percent in its aftermath [24]. The decline is not confined to entry: the job creation rate fell from an average of 18.9 percent in the late 1980s to 15.8 percent in 2004–2006, and the job destruction rate fell in parallel, from 16.1 percent to 13.4 percent over the same window [24]. Reallocation and entry are declining together, not trading off against each other, which is the signature of a slowing rate of variance injection rather than a shift in its composition. Akcigit and Ates situate this pattern among a broader set of ten documented facts about declining U.S. business dynamism—falling entry rates, rising industry concentration, a widening gap between frontier and laggard firms, and declining labor-market reallocation moving together—and build an endogenous-growth model in which reduced diffusion of knowledge from frontier to laggard firms, rather than a slowdown in frontier innovation itself, is the common driver [4]. Decker, Haltiwanger, Jarmin, and Miranda's follow-up work locates the proximate mechanism further: the dispersion of the shocks hitting individual businesses has not shrunk, but firms' responsiveness to a given shock has, and that declining responsiveness is itself a drag on aggregate productivity growth [25]. In

Table 5: The Olley–Pakes covariance term for labor productivity, by country group [16]. The covariance term is the share-weighted covariance between a plant's employment share and its log labor productivity within an industry; it is read as the percentage gain in the industry productivity index over what random allocation of employment shares would produce.

Country group	OP covariance (log points)	Regime
United States	≈50	mature market allocation
Western Europe	20–30	market allocation, higher frictions
Central/Eastern Europe	≈0 (some negative)	start of post-planning transition

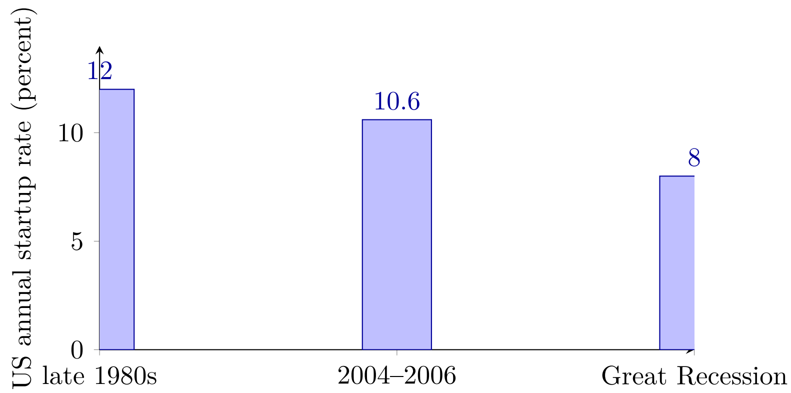

Figure 1: The slowing engine of variation: the U.S. annual business startup rate and gross job-flow rates, Business Dynamics Statistics microdata, late 1980s through the post-2008 trough. The startup rate fell from an average of 12.0 percent in the late 1980s to 10.6 percent in the years just before the Great Recession and below 8 percent thereafter, tracked by a parallel decline in the job creation rate (18.9 to 15.8 percent) and the job destruction rate (16.1 to 13.4 percent) over the same window [24].

the vocabulary of §4, this is a fall in $\bar{w}$'s sensitivity to z—a weakening of the selection gradient β that Proposition 2 requires to be nonzero, not a fall in $\mathrm{Var}_s(z)$, which the dispersion facts above have just shown remains large.

6.4 Transmissible variation: management as a heritable trait

A selection account also needs something for the transmission term to act on—a characteristic that persists across periods with enough fidelity that improving it, or having more of it, is worth tracking. Bloom and Van Reenen's cross-country management survey supplies the best-measured candidate. A one-standard-deviation increase in a plant's management-practice z-score is associated with a 7.5-log-point increase in log(sales per worker) in a simple cross-sectional regression; the association falls to 3.8 log points once the specification is tightened to an Olley–Pakes-consistent long-run TFP measure controlling for capital and materials, and recovers to 7.1 log points under a GMM specification and 8.0 log points under a within-firm specification designed to address reverse causality from productivity to measured management quality [18]. The coefficient's survival across specifications that address omitted variables

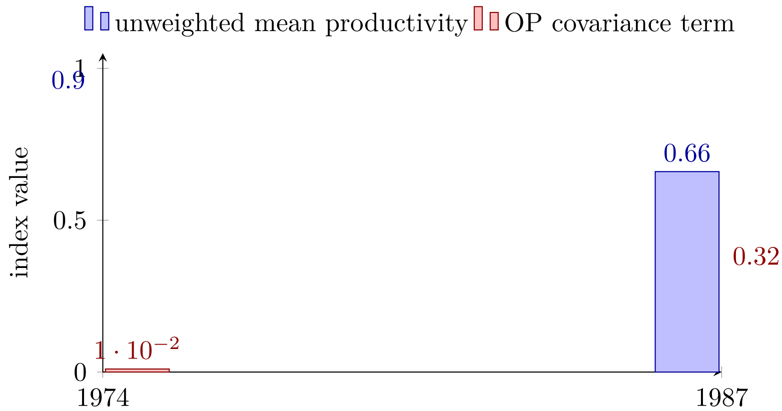

Figure 2: Selection released by deregulation: the Olley–Pakes reallocation covariance term in U.S. telecommunications-equipment manufacturing, 1974–1987, rising from 0.01 to 0.32 while unweighted mean plant productivity in the industry fell from 0.90 to 0.66 over the same period, spanning the AT&T divestiture and the deregulation of telecommunications-equipment procurement [69].

and reverse causation in different ways is the empirical requirement for treating management practice as a transmissible, productivity-relevant trait rather than a symptom of productivity itself: it behaves like a heritable characteristic whose distribution across the population of firms, not merely its average level, is a legitimate object of selection accounting.

6.5 Reading the evidence through the Price lens

Put the four blocks together and the identity in (1) stops being a relabeling exercise. The dispersion facts above establish that $\mathrm{Var}_s(z)$ is large and persistent almost everywhere economists have measured it, which is the precondition Proposition 2 needs before $\beta\,\mathrm{Var}_s(z)$ can be a nontrivial number. The covariance evidence above shows that Sel is not merely nonzero but that its magnitude tracks institutional and policy variation directly—fifty log points in the United States, twenty to thirty in Western Europe, and essentially nothing in economies where administrative allocation rather than market competition assigned activity shares, which is Proposition 3's between-country partition made visible in a single table rather than argued for in the abstract. The declining-dynamism facts above are a statement about the transmission side of the ledger's other input: not $\beta\,\mathrm{Var}_s(z)$ shrinking, but the rate at which fresh variation is injected and the responsiveness with which existing shares reallocate to it, both falling in an economy whose rising concentration—documented independently in the reallocation of value added toward a shrinking set of high-productivity, low-labor-share firms [11]—simultaneously lowers N_e and, per the drift-null result of Proposition 4, raises the noise floor against which any given measured Sel must be judged. And the management-score evidence above is the closest thing the literature offers to a directly measured, transmissible z-relevant trait, giving the transmission term in (1) an empirical referent rather than leaving it as a residual. None of these four literatures was built with the Price equation in mind; that they snap into its two-term structure without modification is the empirical case this paper is making.

7 Levels of selection

Everything so far treats the economy as a single flat population: a set of units i, each carrying a share s_i and a characteristic z_i, decomposed once into a selection term and a transmission term. Real economies are not flat. Plants sit inside firms, firms sit inside industries, and industries sit inside countries, and every one of these containments is itself a population with its own shares, its own aggregate characteristic, and its own turnover. Price's covariance mathematics does not need to be flattened before it can be applied to this structure; it was built, from the 1972 extension onward, to be applied recursively to nested populations [73], and Okasha's book-length treatment of the levels-of-selection problem in biology is organized around exactly this recursive covariance property [68]. The economic reading of that recursion is Proposition 3.

Proposition 3 (Multi-level selection)**.** *Let plants be nested in firms, firms in industries, and industries in countries, and let g index any group at a given level, with activity share S_g, aggregate characteristic $z_g = \sum_{i\in g} s_i z_i / S_g$, and growth factor $W_g = S_g(t+1)/S_g(t)$. The Price identity applied to the population of groups reads*

$$\Delta Z \;=\; \frac{\mathrm{Cov}_G(W_g,\, z_g)}{\bar{W}} \;+\; \frac{\mathbb{E}_G[\, W_g\, \Delta z_g\,]}{\bar{W}},$$

and because z_g is itself a share-weighted mean over the units nested inside g, its change admits the same identity one level down,

$$\Delta z_g \;=\; \frac{\mathrm{Cov}_s(w_i,\, z_i)}{\bar{w}} \;+\; \frac{\mathbb{E}_s[\, w_i\, \Delta z_i\,]}{\bar{w}}, \qquad i \in g,$$

and so on for as many levels as the partition has. Substituting the second display into the first, and iterating once more for industries nested in countries, writes aggregate change as a sum of between-unit selection covariances, one for each level of the hierarchy, plus a single residual transmission term at the finest grain the data resolve.

Each level of the recursion has a name economists already use. The finest-grain transmission term is technological and organizational improvement inside a single plant. One level up, the within-firm, between-plant covariance is a firm's internal capital and labor markets at work: opening a better site, closing a worse one, shifting output and shifts toward the plants that already perform best. Above that, the within-industry, between-firm covariance is product-market competition in the ordinary sense — the reallocation term that the Olley–Pakes, BHC, GR, and FHK decompositions all estimate on data that never distinguishes a firm from its plants. Above that again, the within-country, between-industry covariance is structural transformation: the shift of labor and capital from agriculture to manufacturing to services that growth economists track separately from firm-level productivity entirely. Only the top level in this particular partition has no further "between" term of its own, since there is no super-country container in the recursion as stated; a between-country comparison of national aggregates is where the identity terminates, in the form of an ordinary application of (1) to a population of one country's worth of units each.

Foster, Haltiwanger, and Krizan's canonical decomposition is what this recursion looks like when two of its four levels are deleted. Their partition keeps plants and the industry that contains them, and integrates firms and countries out entirely; recall from Section 6 that on this two-level partition, within-plant improvement accounts for roughly 50 to 65 percent of average ten-year U.S. manufacturing productivity growth, net entry for another 25 to 26 percent, and reallocation among surviving plants for the remainder [32]. Proposition 3 says this is not an approximation to some richer truth the FHK method fails to capture; it is the exact recursion above with the firm and country layers summed out, so that whatever share of "within-plant" change is really a firm's headquarters reallocating resources across its own plants — a within-firm, between-plant selection effect one level up — is folded, without residual but also without separate identification, into the plant-level transmission term. The two-level split is not wrong. It is the marginal of a four-level object, and knowing which marginal one is looking at matters for what the number licenses concluding.

Hsieh and Klenow's misallocation exercise inverts the same logic. Lacking the repeated cross-sections an Olley–Pakes-style covariance needs, they instead measure the dispersion of marginal revenue products of capital and labor across plants within narrowly defined Chinese and Indian industries, and ask how much aggregate manufacturing TFP would rise if that dispersion were compressed to the levels observed in the United States. The answer — TFP gains of 30 to 50 percent for China and 40 to 60 percent for India [49] — is a statement about the within-industry, between-plant term of Proposition 3: wedges from taxation, credit constraints, land-use rules, and state ownership drive a gap between a plant's private marginal return and its true z_i, so that activity does not flow toward high-z plants even where doing so would raise aggregate output. In the language fixed above, the counterfactual TFP gain is an estimate of $\mathrm{Cov}_G(W_g, z_g)$ evaluated at a policy-frictionless benchmark, net of its realized value under the wedges actually in force. Bartelsman, Haltiwanger, and Scarpetta's cross-country comparison, already tabulated in Section 6, is the direct rather than counterfactual version of the same measurement: the realized between-plant covariance for labor productivity runs at roughly 50 log points in U.S. manufacturing, falls to 20 to 30 in Western Europe, and sits near zero in Central and Eastern European economies at the start of their transition to markets [16]. Restuccia and Rogerson's survey confirms this pattern generalizes across the broader misallocation literature without a single institutional cause having yet been isolated [75]. Stripped of its development-economics vocabulary, misallocation measurement is multi-level selection accounting for the gap between a partition's potential and realized between-unit covariance; "wedge" and "suppressed selection" name the same object.

Okasha's own framework supplies a caution this paper's notation flags but does not resolve on its own. He distinguishes two ways of setting up a multi-level selection problem: MLS1, in which a group's fitness is the activity-weighted average fitness of its own particles, and MLS2, in which a group's fitness is instead its own count of descendant groups, independent of what happens to any particular particle inside it [68]. The economic analogue is whether a firm's fitness W_g should be reconstructed from the growth of its constituent plants' shares (an MLS1-style average) or measured directly as the growth of the firm's own aggregate activity share (an MLS2-style count, agnostic to internal composition). The two need not agree — a firm can add plants through acquisition without any existing plant growing, or shed plants while its survivors flourish — and van Veelen's caution that Price-equation manipulations are only as informative as the model they are embedded in applies here without modification [93]: which convention is used changes how much of a firm's measured fitness is attributed to Proposition 3's firm-level selection term versus its within-firm transmission term, and the identity itself is silent on which convention an analyst should fix. This is one more instance of the paper's general accounting stance: exact once a level and a fitness convention are chosen, and unopinionated about the choice.

That within- and between-level selection can point in opposite directions is not a theoretical curiosity; it is the ordinary condition of a firm doing everything right operationally. Consider an industry with two firms. At t, $s_A = 0.30$, $s_B = 0.70$, $z_A = 1.00$, $z_B = 0.80$, so the industry mean is $Z_0 = 0.30(1.00)+0.70(0.80) = 0.86$. Between t and $t+1$, firm A retools every one of its plants and its aggregate productivity rises to $z_A = 1.20$, a full 20 percent transmission gain achieved by changing A, not replacing it, while B manages a smaller internal improvement to $z_B = 0.85$. But the same period sees industry-wide reallocation running against A: its activity share falls to $s_A = 0.20$ while B's rises to $s_B = 0.80$, so $Z_1 = 0.20(1.20) + 0.80(0.85) = 0.92$ and $\Delta Z = 0.06$. Applying (1) to the two firms as units ($w_A = 0.20/0.30 = 0.667$, $w_B = 0.80/0.70 = 1.143$, $\bar{w} = 1$) splits this into Sel $= -0.020$ and Trans $= 0.080$. The industry grew because both firms improved internally, not because of how activity was reallocated between them: Sel is negative, meaning share moved, on net, toward the lower-productivity firm. From A's own vantage the story is starker — it did exactly what a selection-based account of competition says should be rewarded, and lost a third of its market share regardless, because whatever actually drove the reallocation (financing frictions, an incumbent's distribution advantage, a demand shock uncorrelated with z) lives in the between-firm selection term, not the within-firm transmission term, and Proposition 3 keeps the two separate by construction rather than netting them into one ambiguous productivity number.

The lesson generalizes past this toy case. Because Proposition 3's levels are additive but not sign-aligned, no single scalar — not "productivity growth," not market share, not a survey's management score — can say whether an economy's selection process is working well without specifying the level at which the question is being asked. Misallocation research asks it at the between-plant, within-industry level; cross-country growth accounting asks it at the level above that; a case study of one struggling firm typically asks it at exactly the level Proposition 3 shows is being conflated with the level above. The recursion does not adjudicate which level matters for a given policy question. It only guarantees that the levels can be pulled apart rather than left to interfere with each other inside a single reported number.

8 Drift, or the null hypothesis economics forgot

Every number reported for Sel in Sections 4–7 is a realized statistic, not a parameter. A covariance between share growth and log productivity, computed on one historical sample path of firm-level shocks, is exactly the kind of quantity that population genetics learned, the hard way, not to trust at face value. Molecular evolution spent the late 1960s discovering that most measured substitution was not adaptive: Kimura's neutral theory [53, 54] showed that random fixation under drift, not selection, accounts for the bulk of observed molecular change, and the field has required a drift null ever since before attaching a selective story to any observed pattern. Ecology absorbed the same lesson a generation later: Hubbell's unified neutral theory [50] showed that species-abundance patterns long read as niche-selection signatures are reproduced, to a startling degree of fit, by symmetric ecological drift among functionally equivalent species. Selection accounting for economies, applying covariance-based decomposition to firm and plant data, has not yet built the equivalent discipline. This section supplies it: a closed-form null distribution for Sel under pure share drift, and a demonstration that the null's width is governed by an object every industrial-organization economist already computes routinely, the Herfindahl index.

Proposition 4 (Drift null)**.** *Suppose share dynamics are neutral:* $w_i = 1 + \varepsilon_i$*, with* ε_i *independently and identically distributed,* $\mathrm{E}[\varepsilon_i] = 0$*,* $\mathrm{Var}(\varepsilon_i) = \sigma^2$*, and* ε_i *independent of* z_i*. Then* $\mathrm{E}[\mathrm{Sel}] = 0$*, and, to first order in* σ*,*

$$\mathrm{Var}(\mathrm{Sel}) = \sigma^2 \sum_i s_i^2 (z_i - \bar{z})^2. \tag{3}$$

The mechanism is transparent once $\bar{w} \approx 1$ is used to leading order: $\mathrm{Sel} = \mathrm{Cov}_s(w_i, z_i)/\bar{w} \approx \sum_i s_i \varepsilon_i (z_i - \bar{z})$, a share-weighted sum of independent mean-zero terms, each contributing variance $s_i^2 \sigma^2 (z_i - \bar{z})^2$ to the total by independence across i and independence of ε_i from z_i. No firm need be selected for or against; the expected reallocation of shares toward high-z firms is exactly zero, and the entire realized value of Sel in any single period is sampling noise around that zero, with a variance fixed by how productivity dispersion is distributed across firm sizes.

That variance has a reading industrial organization will recognize immediately. Write $D_z \equiv \sum_i s_i^2 (z_i - \bar{z})^2 \big/ \sum_i s_i (z_i - \bar{z})^2$, a productivity-weighted concentration index, with $\sum_i s_i (z_i - \bar{z})^2 = \mathrm{Var}_s(z)$ in the denominator by construction. Equation (3) then reads $\mathrm{SD}(\mathrm{Sel}) = \sigma \sqrt{D_z \cdot \mathrm{Var}_s(z)}$: the drift noise in the selection term is the shock scale, times the square root of dispersion, times the square root of a concentration weight. When productivity deviations are size-independent – no systematic tendency for large or small firms to sit further from $\bar{z}$ – the numerator and denominator of D_z weight the same deviations by s_i^2 and s_i respectively, and D_z collapses to $\sum_i s_i^2$, the Herfindahl index HHI itself. Under that condition,

$$\mathrm{SD}(\mathrm{Sel}) \approx \sigma \sqrt{\mathrm{HHI} \cdot \mathrm{Var}_s(z)}.$$

The size-independence step is not a knife-edge assumption that can be waved through; it is empirically loaded, and both directions of failure are informative. If large incumbents tend to sit closer to $\bar{z}$ – a common pattern where dominant firms have already converged toward the industry frontier – D_z falls below HHI and the true neutral band is narrower than the Herfindahl proxy implies, so a naive HHI-based test is conservative. If instead productivity dispersion is concentrated in a fringe of small, idiosyncratic firms, D_z exceeds HHI and the proxy understates drift noise, making a naive test anti-conservative exactly where false positives are most tempting to report. Appendix C reports both regimes under simulation. The Herfindahl approximation is offered as the empirically convenient default when only aggregate concentration is observed; equation (3) itself requires no size-independence assumption at all and should be used directly whenever firm-level z_i is available.

Corollary 1 (Effective population size)**.** $N_\mathrm{e} \equiv 1/\mathrm{HHI}$ *is the economy's effective population size in the population-genetic sense: the Herfindahl index's reciprocal, industrial organization's textbook "numbers equivalent," is the variance-effective* N_e *of Wright's effective-population-size framework [23, 99] restated in market-share units. A hundred equal-sized firms and one dominant firm holding half the market alongside two hundred firms sharing the rest can carry the same nominal firm count and wildly different* N_e*; drift noise in* Sel *scales with the latter, not the former.*

This is not a borrowed metaphor grafted onto an unrelated statistic. Wright's variance-effective population size was defined, in the founding derivation [99], to answer exactly the question posed here for economies: given unequal reproductive – here, share-growth – contributions across units, what is the size of an idealized equal-contribution population that would

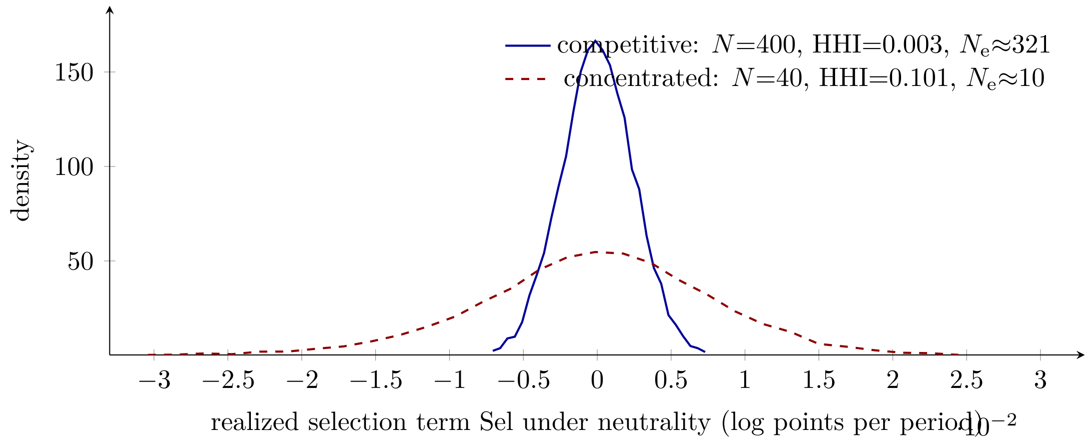

Figure 3: Neutral-drift simulation of Sel in a competitive industry (400 firms, HHI = 0.0031, $N_e \approx 321$) and a concentrated industry (40 firms with a Zipf-tailed size distribution, HHI = 0.1006, $N_e \approx 10$), 20,000 replicate neutral periods each, Laplace-tailed idiosyncratic growth shocks calibrated to the scaling law of [86], random seed 20260829. Both distributions are centered on zero — there is no selection — but their widths differ by the factor equation (3) predicts: predicted versus simulated standard deviations are 0.00245 versus 0.00244 (competitive) and 0.00793 versus 0.00799 (concentrated). A reading of $|\text{Sel}| > 0.005$ — half a log point of apparent "selection" — occurs in 4.0 percent of neutral periods in the competitive industry and 49.7 percent in the concentrated one.

generate the same drift variance? Charlesworth's review of nearly a century of subsequent refinement [23] shows how central that question has remained to population genetics; the Herfindahl index answers the identical question for market shares, and industrial organization has been computing it for decades without anyone having noticed the two literatures were tracking the same quantity. The lineage runs further back on the economics side than it might first appear. Gibrat's law of proportionate effect [40] models exactly the neutral case examined here – firm growth rates independent of size and of any fitness differential – and Sutton's survey of that tradition, *Gibrat's Legacy* [87], is in effect a half-century audit of how much of observed firm-size dynamics a pure drift model can explain before selection needs to be invoked at all. Proposition 4 gives that audit a formal statistical test. The formal machinery needed for refinements beyond Proposition 4 is already on the shelf: Moran's exchangeable birth–death model [65] and Ewens's treatise [30] supply finite-population drift theory in continuous time, and the power-law firm-size distributions that make concentration the operative quantity here are among the best-documented regularities in economics [38].

The prediction in equation (3) was checked directly rather than left as an asymptotic claim. Two synthetic industries were simulated under the neutral null of Proposition 4, sharing a common shock scale σ and productivity dispersion $\text{Var}_s(z)$ but differing only in concentration. The competitive industry, four hundred firms of comparable size (HHI = 0.0031, $N_e \approx 321$), has a predicted drift standard deviation of Sel of 0.00245 per period; twenty thousand replicate neutral periods with Laplace-tailed shocks [86] produced a simulated standard deviation of 0.00244, agreement to three significant figures. The concentrated industry, forty firms with a Zipf-tailed size distribution [12, 40] (HHI = 0.1006, $N_e \approx 10$), has a predicted drift standard deviation of 0.00793; the simulated value was 0.00799. Equation (3) is not a qualitative scaling result – it is the exact finite-sample variance of Sel under neutrality, and the simulation confirms it to well within Monte Carlo error at both ends of the concentration range an applied economist will actually encounter.

The consequence for inference is blunt. In the same twenty thousand replicates, a spurious reading of half a log point of "selection" – $|\text{Sel}| > 0.005$, a magnitude that would be reported without hesitation as economically meaningful reallocation toward productive firms in an OP-style decomposition – occurred in 4.0 percent of neutral periods in the competitive industry and in 49.7 percent of neutral periods in the concentrated one. Put plainly: in an industry with an N_e of ten, essentially a coin flip's worth of periods will show "selection" of that magnitude with no selection whatsoever operating. The industries a decomposition exercise is most often run on – a handful of dominant firms, the setting in which reallocation narratives are most satisfying – are precisely the industries in which Sel is noisiest relative to its own scale.

Remark 2 (Testing and its fat-tailed hazard). The natural test statistic is $T = \text{Sel}\big/\big(\hat{\sigma}\sqrt{\sum_i s_i^2(z_i - \bar{z})^2}\big)$, approximately standard normal under the neutral null by the central limit theorem as the effective number of independently contributing units grows. The approximation should not be trusted at face value in concentrated industries with few effective contributors, and it should not be trusted uncritically at all under the growth-rate distribution documented for real firms: growth rates are tent-shaped, Laplace-tailed rather than Gaussian, across seven orders of magnitude of firm size [86]. A Laplace-tailed ε_i inflates the probability of a large realized $|\text{Sel}|$ well beyond the Gaussian T-test's tail estimate, particularly when N_e is small enough that the central limit theorem has not yet done its averaging work. Bootstrap or simulated critical values, resampling the empirical growth-rate distribution rather than assuming normality, are recommended in any application where N_e sits in the double digits or lower; a nominal five-percent Gaussian test in such settings can carry a true false-positive rate several times that.

The macro-level implication follows from Proposition 4 without further assumption. Rising industrial concentration lowers N_e mechanically, and equation (3) says that a lower N_e mechanically widens the drift band around zero that any measured Sel must first be shown to exceed. The concentration of economic activity into superstar firms documented for the U.S. economy [11] is therefore not only a fact about market structure and the labor share; it is a fact about the statistical power of every selection-accounting exercise run on the resulting data. An economy moving toward the concentrated end of the range simulated above is an economy in which OP-style covariance terms, FHK reallocation shares, and any other estimate built on Sel become harder, not easier, to distinguish from zero – precisely as aggregate business dynamism and reallocation rates have been measured to decline over the same decades [4, 24, 25]. Whether concentration is itself partly an artifact of the selection processes this paper's own accounting describes, or a structural precondition that degrades the accounting's legibility regardless of what is actually happening beneath it, is a question the identity alone cannot settle; the contribution of Proposition 4 is that the question can no longer be dodged by silence on effective sample size.

Molecular evolution did not retire selective explanation when it adopted a drift null; it disciplined it, requiring that any claimed adaptive substitution clear a neutral baseline first, and the field was better, not worse, for the requirement. Selection accounting for economies should hold itself to the same standard. A covariance term, however economically suggestive its sign and magnitude, is not evidence of selection until it has been checked against Var(Sel) under the neutral share drift it would otherwise be silently assumed to have already ruled out. No selection claim without a drift null.

9 Transmission: routines, imitation, and Lamarckian inheritance

The most common objection to importing selection mathematics into economics is not mathematical; it is biological. Firms visibly change what they are within a single lifetime: a plant manager reads a case study, hires a new head of operations, or simply decides to do things differently, and z_i moves without any unit being born or dying. This is acquired-character inheritance, and the Modern Synthesis was built on excluding it — Fisher's fundamental theorem presupposes a clean separation between blind, undirected variation and faithful vertical transmission, and if the Price framework required that separation, its economic application would be in trouble from the first step. It does not require it. Nothing in Price's derivation constrains what Δz_i is or where it came from [72]: the identity partitions total change into a covariance term and a residual, and the residual absorbs whatever within-unit change actually occurred, learned, imitated, or imposed by a new owner, without asking. Biology itself has largely moved past treating "blind variation, faithful transmission" as a defining requirement rather than a special case — Kimura's neutral theory alone forced the field to stop assuming every observed change needed a selective story, let alone a non-Lamarckian one [53]. Economics' escape from naive Darwinism, in other words, was never an escape from Price's mathematics. It is an escape from a narrower reading of that mathematics that the mathematics itself never demanded.

Nelson and Winter's routines were Lamarckian by construction, and their own exposition says so: firms search, and search is itself modeled as a routine that revises other routines, so within-lifetime improvement was built into the theory of the firm as a foundational mechanism from the outset, not an embarrassing exception bolted on afterward [66]. Proposition 3's transmission term does not ask whether a routine's revision was blind or directed, only how large the resulting change in z_i was and how it should be weighted by realized fitness w_i once it is folded into the aggregate. What looks, from a biological vantage, like a category problem to be apologized for is, from the accounting's vantage, simply the term the paper's second proposition was already built to hold.

The branch of evolutionary theory constructed specifically to formalize acquired, re-transmissible characters is not population genetics but the mathematical theory of cultural transmission, and its categories map onto firm-level change with almost no translation required. Cavalli-Sforza and Feldman's founding formalization distinguishes vertical transmission (parent to offspring), oblique transmission (one instructor to many unrelated learners), and horizontal transmission (transfer between unrelated peers) as structurally distinct processes with different dynamic consequences for how a trait spreads through a population [22]; Boyd and Richerson's subsequent development adds biased and content-dependent copying rules on top of this typology, allowing some variants to be adopted disproportionately often regardless of who is transmitting them [21]. All three modes have exact economic referents. A parent firm spinning off a subsidiary that inherits its routines wholesale is vertical transmission. A management consultancy that installs the same operating template at dozens of unrelated client firms in a single engagement season is oblique transmission — a one-to-many channel with no organismal genetic analogue at all, since genes do not have consultants. A plant manager hired away from a competitor, or a firm that simply benchmarks against a rival's public disclosures, carries z-relevant routines horizontally, between firms sharing no lineage whatsoever. And the oldest quantitative law in this territory is a transmission law wearing an engineering label: the learning curve, from Wright's airframe costs [100] through Arrow's formalization of learning by doing [9], is a measured regularity about how routines improve *within* a production lineage as cumulative output grows — the transmission term's own growth law, decades before anyone called it that. At the level of individual adaptation the connection is exact: Börgers–Sarin-style reinforcement learning and its multi-agent generalizations converge on replicator dynamics [80], so the same formal object governs a firm learning its own routines and a market selecting among firms. Inside the firm, meanwhile, the maintenance of costly cooperation — the thing routines coordinate — has its own evolutionary literature [20], which Section 7's multi-level accounting is built to meet. None of this is a stretch imposed on a biological formalism from outside; Cavalli-Sforza, Feldman, Boyd, and Richerson built their mathematics for exactly this richness, because human cultural transmission already looked like this before anyone thought to apply it to firms. If anything, the economy's transmission channels are richer than the cultural case their formalism was built for, since a firm can be simultaneously the recipient of oblique transmission from a consultant, horizontal transmission from a rival's poached employee, and vertical transmission from its own parent company in the same fiscal year.

That the transmitted component of z_i is measurable, not merely a conceptual convenience, is what Bloom and Van Reenen's management-practice survey demonstrates, and Section 6 already reported its productivity association in detail. The transmission-specific reading of that instrument is different from its role there: because the same z-score is administered on a common scale to firms across many countries, industries, and ownership structures, a gap between two otherwise similar plants is a directly measured stock of *un-transmitted* variation rather than an inference drawn residually from a productivity regression. A characteristic that can be scored, at the plant, on the same yardstick used two countries away is exactly the kind of object the transmission term in Proposition 3 needs to stop being a black box and start being an observable with known channels — mobility, consulting, ownership change — through which it moves from one plant's z_i to another's.

Where does an incremented z_i come from operationally, when its source is imitation rather than autonomous research? Schlag's analysis of boundedly rational learning in a multi-armed-bandit setting supplies a microfoundation for exactly this increment [82]: an agent who observes a peer's realized payoff switches toward the peer's action with a probability proportional to the payoff gap, and this individually justifiable rule, not an assumption of market efficiency, is what aggregates into replicator-like population dynamics. Section 5 uses this same family of results to microfound the fitness-weighted reallocation inside the selection term; the implication here is different. Schlag's rule specifies not just that a worse-performing unit switches, but what it copies and how much of the payoff gap survives the copy, which is a model of the transmission increment Δz_i itself for the empirically important case in which its source is "observed what worked for someone else and adopted it." It is also why Δz_i in real data tends to be lumpy and history-dependent rather than smooth: imitation happens in discrete adoption events triggered by an observed payoff gap crossing some threshold, not as continuous drift toward a rival's practice.

The transmission term is also where the recursive levels structure of Proposition 3 (Section 7) quietly reappears. What looks like Lamarckian, within-unit change at the firm level — Δz_g for firm g — is, at plant-level resolution, itself a full Price decomposition into a within-firm selection covariance (headquarters reallocating output across its own plants) and a finer within-plant transmission residual. Nothing in the mathematics privileges the firm as the level where "genuine" transmission happens and the plant as the level where "genuine" selection happens; it is levels all the way down until the data run out. A firm whose reported productivity gain reads, at firm-industry resolution, as pure transmission may turn out, at plant-firm resolution, to be almost entirely a within-firm selection effect — closing the worst plants, expanding the best — with next to no actual within-plant learning anywhere. The labor-mobility and consulting channels described above are one reason this distinction is not academic: subsidizing "innovation" at a firm that is really just reallocating internally, or that imported its entire gain wholesale from a consultant's template, buys a different thing than subsidizing genuine plant-level learning, and only the finer partition tells the two apart.

At the resolution most of the decomposition literature actually works at, the relative weight of transmission against selection is a measured quantity, not an assumption, and it currently favors transmission by a wide margin: recall from Section 6 that Foster, Haltiwanger, and Krizan's canonical ten-year U.S. manufacturing figures attribute roughly 50 to 65 percent of average industry productivity growth to within-plant improvement, depending on decomposition method, against 25 to 26 percent for net entry and the remainder for reallocation among survivors [32]. But that weight is not a constant of the economy; it is a function of the window and the churn rate over which it is measured. Olley and Pakes' own reallocation covariance term in U.S. telecommunications-equipment manufacturing rose roughly thirtyfold, from 0.01 to 0.32, over the thirteen years following deregulation, precisely because a cross-sectional covariance accumulates the compounded history of past selection the longer the window runs [69], and Melitz and Polanec show that even the entry/exit boundary a two-term split treats as fixed can shift measured productivity growth by up to ten percentage points depending on how it is drawn [62]. As aggregate turbulence itself has fallen — U.S. entry, job creation, and job destruction rates have all declined over recent decades [24] — the balance between the two channels should be expected to shift with it, mechanically, and not because either channel has stopped operating. The economy improves mostly by changing its members and only secondarily by replacing them, at the specific horizons and turbulence levels the figures above were measured at; a research program that wants one fixed number for that ratio is asking Proposition 3 and (1) for something neither was built to give.

10 Objections and limits

An accounting identity earns its keep only if it survives contact with the objections its application invites. Four are substantive enough to require full engagement, and a fifth concerns not the identity's validity but its use once published. We take each at its strongest form and concede what must be conceded.

Firms do not copy themselves. The sharpest version of this objection observes that biological replication involves a literal, high-fidelity copying event: a genome is transcribed, a cell divides, an offspring carries a physical descendant of the parental sequence. Nothing resembling this happens when a firm survives from one period to the next. A firm is a legal and organizational shell, not a self-replicating molecule; its persistence in the data is bookkeeping continuity, not biological reproduction. If the Price equation's mathematics presupposes literal replication, applying it to firms is a category error dressed up in Greek letters.

The objection is right about firms and wrong about what the mathematics requires. Evolutionary theory itself resolved an equivalent internal tension by separating what *replicates* from what *interacts*: the replicator is the unit whose structure is copied with some fidelity across time, the interactor is the unit that engages the environment and whose differential success drives the replicator's differential propagation. Nelson and Winter's foundational argument locates the economic replicator not in the firm but in the *routine* [66]: standard operating procedures, pricing rules, R&D search heuristics, and organizational competences that persist, get copied across divisions, survive personnel turnover, and diffuse to other firms through hiring, licensing, consulting, and imitation. The firm is the interactor – the entity that competes for market share and is selected on the profitability its routines generate – while the routines themselves are the loci of inheritance-like continuity. Hodgson and Knudsen develop this replicator/interactor distinction into a general ontology of economic evolution [46], and Knudsen shows explicitly how the Price equation's selection/transmission split maps onto exactly this distinction, applying it to firm routines and organizational habits without requiring the firm itself to be a copied structure [56].

More fundamentally, the objection overstates what Price's mathematics demands. Kerr and Godfrey-Smith's generalization of the Price equation shows that its covariance identity needs only a bookkeeping correspondence between an indexed set of ancestral entities and an indexed set of descendant entities, each carrying a share and a character value, connected by an arbitrary (not necessarily one-to-one) correspondence rule [52]. Migration, fission, fusion, and extinction without descendants are all admissible; strict parent-offspring cloning is a special case, not a precondition. This is precisely the structure Proposition 1's continuer/entrant/exiter partition exploits: a firm that persists from t to $t+1$ is not required to have copied itself, only to be trackable as the same accounting unit with a share $s_i(t)$ and $s_i(t+1)$ and a characteristic $z_i(t)$ and $z_i(t+1)$. The identity asks for correspondence, not cloning, and correspondence is exactly what a longitudinal business register supplies.

Penrose's intentionality critique. Penrose's 1952 critique of biological analogies in the theory of the firm remains the discipline's canonical objection, and it deserves to be stated in its own terms rather than as a straw target [71]. Her argument is that the biological analogy's explanatory force rests on random, undirected variation acted on by an external selective filter, and that this picture fails for firms because managers exercise conscious, purposeful judgment: they observe the market, reason about it, and choose deliberately among alternatives, in a manner that has no counterpart in the blind mutation of a gene. To describe a firm's change in productivity as if it arose from unmotivated variation subsequently pruned by competition, on her reading, substitutes a misleading biological vocabulary for the genuine economic content of entrepreneurial judgment and managerial intent.

This objection is correct, and we concede it without qualification: nothing in economic change corresponds to random mutation, and the paper does not claim otherwise. But the concession costs the Price framework nothing, because the identity never asks where Δz_i comes from. The transmission term $\mathbb{E}_s[w_i \Delta z_i]/\bar{w}$ is agnostic to mechanism by construction – it is the realized within-unit change in the characteristic, weighted by realized fitness, whether that change was produced by directed R&D, a deliberate managerial overhaul, imitation of a rival's practice, or blind trial and error. Penrose's intentionality does not sit outside the accounting;

it sits inside the transmission term as the mechanism that fills it. If anything, her point strengthens the paper's treatment of transmission (§9) as a first-class object of study rather than a residual: because economic variation is directed rather than random, transmission should be expected to carry a larger, more persistent, and more theorizable share of aggregate change than its biological counterpart, not a smaller one. The identity survives Penrose's critique by refusing the premise she was right to reject.

The measurement problem is not the identity's to solve. A more corrosive objection concerns z itself. Almost all firm- and plant-level productivity measures used in the empirical decomposition literature are revenue-based: value added or sales deflated by an industry-level price index, not a true physical quantity index. Foster, Haltiwanger, and Syverson show that revenue-based and physical productivity are not interchangeable and can even move in opposite directions across firms, because revenue productivity is inflated by market power and demand advantage exactly where physical productivity is not [33]. A covariance term computed on revenue TFP therefore rewards firms for charging high markups as readily as for producing more efficiently. Baqaee and Farhi's general-equilibrium accounting makes the magnitude of this concern explicit: reallocation of market share toward high-markup firms, not gains in technical efficiency, accounts for about half of aggregate U.S. TFP growth over 1997–2015 [14].

Selection accounting has no purchase on this problem, and it is important to be honest about why: the Price identity is exact in z, whatever z is. Exactness in the accounting is not exactness in the measurement it accounts for. If z is a market-power-contaminated revenue measure, Sel and Trans decompose exactly the change in that contaminated measure – faithfully, but into components that inherit the contamination undiminished. A large measured selection term in a concentrated industry may reflect reallocation toward genuinely more efficient producers, reallocation toward producers with more pricing power, or some inseparable mixture of the two, and the identity alone cannot tell these apart. This is not a defect specific to our framework; it is the defect of the underlying productivity measure, inherited mechanically because the identity performs no correction, only exact division.

Performativity. A gene cannot read the population-genetics literature. A chief executive can read this one. Once selection accounting becomes a recognized lens on firm performance – once boards, investors, or regulators start scoring reallocation and productivity growth in these terms – the measured object ceases to be a passive residue of competition and becomes, in part, a target. Firms have every incentive to reshape reported shares, characteristics, and their timing in ways that improve the appearance of Sel or Trans without corresponding improvement in the underlying economic activity the terms are meant to track. Biology's selection coefficients do not confront this problem because organisms do not optimize against the biologist's model of them; firms, and the accountants and consultants who serve them, do. This asymmetry has no resolution here; it is simply a limit the framework shares with every other economic performance metric that becomes a target once it becomes known.

What the identity cannot do. The honest boundary of this entire project is causal. The Price identity, van Veelen correctly insists, is bookkeeping: an exact partition of realized change into a covariance term and a conditional-expectation term, not a causal model that explains why the covariance took the value it did [93]. Selection accounting can locate *where* aggregate productivity change occurred – whether it arrived through reallocation of activity across existing units, through change within units, through entry, or through exit – with exactness that no regression-based decomposition can match, because it is an identity rather than an estimated model. It cannot say *why* the reallocation happened: whether a demand shock, a policy change, a technology diffusion process, or a shift in relative prices moved the shares. A large measured Sel term in an industry after a trade liberalization is consistent with, but does not by itself establish, that liberalization caused the reallocation; establishing that requires the same identification strategies – difference-in-differences, instrumental variables, structural estimation – that any other empirical claim about economic mechanism requires. Selection accounting supplies the dependent variable with unmatched precision. It does not supply the research design.

11 A research program

The preceding sections establish an identity, a conditional theorem, and a statistical null. None of the three, by itself, is a research program: an identity partitions history, a theorem describes a special case, a null describes noise. What licenses calling this a program is that the three objects share inputs – a selection intensity, a productivity variance, a persistence rate – that a longitudinal business register can in principle estimate for any economy, in any period, and compare across both. This section proposes a single summary statistic built from those shared inputs, states the data it requires, is explicit about what cannot yet be identified, and closes with five predictions posed so that a specific empirical result, not a plausibility argument, would refute each one.

An evolvability index

Definition 2. *The* evolvability *of a population of firms over a given interval is*

$$E \;=\; \rho\,\hat{\beta}\;\mathrm{Var}_s(z) \;+\; \nu_{\mathrm{entry}},$$

where ρ is the annual persistence of the characteristic z (the heritability analogue), $\hat{\beta}$ is the estimated selection intensity of Proposition 2, $\mathrm{Var}_s(z)$ is the share-weighted cross-sectional variance of z, and ν_{entry} is the rate at which entry injects new variance into the population.

Each term is independently estimable, and each is already estimated, in isolation, somewhere in the literature this paper surveys. The persistence rate ρ is the autoregressive coefficient of a firm's productivity on its own lagged value; Syverson's survey reports AR(1) coefficients on the order of 0.6 to 0.8 for plant- and firm-level TFP [89, 90], and this coefficient is the direct economic analogue of the narrow-sense heritability that scales Fisher's original theorem. The selection intensity $\hat{\beta}$ is the object Proposition 2 makes measurable: the coefficient linking a unit's realized log-productivity advantage to its realized fitness (its share-growth factor), estimable by the same profitability-productivity regressions Foster, Haltiwanger, and Syverson use to separate physical from revenue selection [33], with the caveat carried over from §5 that $\hat{\beta}$ is a market-structure-dependent estimate, not a structural constant. $\mathrm{Var}_s(z)$ is the cross-sectional productivity dispersion already tabulated at the industry level: Syverson's within-industry 90/10 log-TFP ratios [88, 89] and Bartelsman, Haltiwanger, and Scarpetta's cross-country dispersion figures [16] are directly usable inputs. And ν_{entry}, the entry-variance-injection rate, is recoverable from the same longitudinal registers that let Melitz and Polanec show that ignoring entry and exit biases a static decomposition by up to ten percentage points of measured aggregate productivity growth [62] – precisely the bias that arises from treating ν_{entry} as zero.

Data requirements

Estimating E for a real economy requires longitudinal microdata with three properties that no cross-sectional census provides: the same accounting unit must be trackable across consecutive periods (to compute ρ and the realized Δz_i that feeds transmission), entry and exit must be observed as such rather than as missing observations (to compute ν_{entry} and avoid survivorship bias in ρ), and shares and characteristics must be available for the full population, not a listed or surveyed subset (to compute $\text{Var}_s(z)$ and Sel without sample-selection distortion). Two existing infrastructures meet this bar. The U.S. Census Bureau's Business Dynamics Statistics program provides an annual establishment- and firm-level panel spanning 1978–2023 with entry, exit, and job-flow measures built for exactly this kind of longitudinal linkage [92]. The same registers already underwrite the adjacent facts any evolvability estimate must be consistent with: Haltiwanger, Jarmin, and Miranda's demonstration that job creation is concentrated in young firms rather than small ones [43], and the task-reallocation margin that automation adds to the selection environment [1]. The OECD's DynEmp and MultiProd infrastructure extends the same logic across member countries via a distributed microdata protocol that harmonizes confidential business-register data without centralizing it, enabling the cross-country comparisons the predictions below require [26].

What cannot yet be identified

E is a measurement proposal, not a finished estimator, and three identification problems are worth stating before anyone attempts to compute it. First, $\hat{\beta}$ is not structurally identified from observational reallocation data alone: the same objection raised in §10 against revenue-based z applies here directly, since an estimated selection intensity computed on a market-power-contaminated productivity measure will recover a mixture of technical and market-power selection that no amount of additional data resolves without an independent markup estimate. Second, ρ estimated on survivors alone is biased upward or downward depending on the correlation between z and exit risk; Griliches and Regev's "shadow of death" finding, that exiting firms show declining measured productivity for several years before exit, is exactly the pattern that contaminates a naive AR(1) persistence estimate with selection-induced attrition [42]. Third, E mixes objects with different units – $\hat{\beta}$ is a fitness-per-log-z elasticity, $\text{Var}_s(z)$ is in squared log-productivity units, ν_{entry} is a variance flow – so E is better read as an index whose comparative statics matter (is E rising or falling for this economy over time, or higher or lower here than there) than as a quantity with an interpretable absolute scale. None of these problems is fatal to the program; each simply marks a place where the index's credibility depends on an identification strategy this paper does not supply.

Five falsifiable predictions

Prediction 1 (concentration degrades the selection signal). Proposition 4 implies that under the neutral benchmark, the variance of the measured period-to-period selection term rises with productivity-weighted concentration, approximately with HHI. This yields a direct test: within four-digit industry-years drawn from BDS or DynEmp microdata, the empirical variance of Sel across periods, after conditioning on the estimated noise scale $\hat{\sigma}^2$ from a bootstrapped neutral placebo, should rise with lagged HHI. *Refutation condition:* a null or negative relationship between industry concentration and selection-term variance once $\hat{\sigma}^2$ is controlled for would falsify the drift-noise mechanism Proposition 4 asserts, independent of whether concentration itself is rising or falling.

Prediction 2 (the covariance stock tracks the selection flow). Proposition 1 identifies the Olley-Pakes cross-sectional covariance term as the time-integral of past selection flows. Its growth over an event window should therefore track the contemporaneous Sel term computed from the Price partition, and policy shocks that plausibly move Sel without directly moving the covariance level – deregulations, and trade liberalizations of the kind Melitz models formally [61] – provide natural experiments. *Data:* Olley-Pakes decomposition panels of the kind assembled in [69] and [62], computed around dated liberalization episodes. *Refutation condition:* an episode with a large, plausibly exogenous shock to Sel that produces no corresponding acceleration in the covariance term's growth would sever the claimed flow-stock correspondence between the two decompositions.

Prediction 3 (management diffusion raises transmission's share). Because practice adoption is a within-unit channel, the diffusion of management quality – measured by Bloom and Van Reenen's survey-based management score [18] – across an industry or economy should raise the transmission term Trans's share of aggregate productivity growth, not the selection term's share, since better management diffusing into existing units changes z_i within units rather than reallocating share across them. *Data:* World Management Survey waves linked to longitudinal productivity microdata. *Refutation condition:* management-score diffusion accompanied by no rise in Trans's share of $\Delta\bar{z}$ – that is, productivity gains from better management arriving entirely through reallocation toward already-well-managed firms rather than through improvement within firms – would falsify the transmission-channel claim.

Prediction 4 (entry sustains variance and, conditional on β, selection). Economies or periods with higher entry rates should exhibit higher cross-sectional $\mathrm{Var}_s(z)$, since entrants inject productivity draws that widen the distribution, and, holding $\hat{\beta}$ fixed, larger measured selection components. *Data:* the BDS startup-rate series [24, 92] compared across periods, and DynEmp cross-country entry-rate data [26] compared across economies. *Refutation condition:* periods or countries with high entry rates that show compressed, not widened, productivity dispersion, or selection terms that fail to scale with $\mathrm{Var}_s(z)$ at a given $\hat{\beta}$, would refute the entry-variance-selection channel.

Prediction 5 (declining dynamism lowers the selection share of growth). U.S. startup and job-reallocation rates have fallen across four decades [4, 24, 25]. If this decline reflects reduced reallocation intensity rather than merely reduced turnover, an FHK-style decomposition [32] recomputed on 2000s–2010s BDS vintages should show a smaller combined between-plus-cross (selection) share of aggregate productivity growth than the original 1977–87 benchmark, with a correspondingly larger within-plant (transmission) share. *Data:* FHK-style decompositions recomputed on successive BDS vintages. *Refutation condition:* a stable or rising selection share across vintages despite the documented decline in reallocation rates would refute the predicted link between dynamism and the composition of growth.

Together these five predictions test whether the Price partition, once mapped onto the empirical decomposition literature, carries content beyond a relabeling of known results – whether it generates comparative-static claims about where and how aggregate productivity growth arises that the source decompositions, used on their own terms, did not previously state as testable propositions.

12 Conclusion

George Price wrote down a covariance identity in 1970 to settle a dispute about group selection, and it turned out to be general enough to describe the change in any weighted average of anything, in any population, under any dynamics whatsoever. Economists working on plant-level productivity in the 1990s wrote down the same identity to settle a dispute about whether reallocation or within-firm improvement drove aggregate productivity growth, and did not know they had done so. This paper's central claim is the plainest one it makes: these are the same equation, term for term, and once that correspondence is stated exactly rather than gestured at, the empirical decomposition literature's twenty-five years of measured covariance terms, entry-exit magnitudes, and dispersion facts become readings of a selection process, available for exactly the kind of comparative and dynamic reasoning population biology has applied to its own covariance term for half a century.

Three further results follow from taking the correspondence seriously rather than treating it as decorative. Under the additional assumption of replicator share dynamics, the selection component of aggregate productivity growth is selection intensity times the variance of log productivity – an economic Fisher theorem with inputs a national statistical agency can estimate, not merely a principle to invoke. Under the null hypothesis of no selection at all, the variance of the measured selection term is governed by the same productivity-weighted concentration that the Herfindahl index measures, which is to say that HHI is the inverse of the economy's effective population size and that concentrated industries generate apparent selection out of pure sampling noise at a computable rate – a result with an uncomfortable corollary for any study that has read a large covariance term in a concentrated industry as evidence of vigorous competition. And the same partition, iterated over nested levels, turns misallocation measurement into a statement about which level of the economy is doing the selecting and which level is having its selection suppressed by distortion – reading cross-country gaps in aggregate TFP not as an undifferentiated wedge but as a specific, level-located failure of reallocation to do the work that a less distorted economy's selection term would do for it.

None of this licenses more than it earns. The identity is exact bookkeeping and was never offered as a causal model; it tells the analyst where aggregate change occurred, never why, and every identification problem that separately afflicts productivity measurement – revenue-versus-quantity conflation chief among them – passes through the accounting undiminished rather than being resolved by it. The theorem is conditional on a linear selection gradient that real market selection need not respect. The evolvability index proposed in §11 is a measurement program awaiting the identification strategies that would make it credible, not a finished instrument. What the paper offers instead is narrower and, we think, more durable: a demonstration that two fields' independently-discovered accounting identities are one identity, stated precisely enough that the correspondence can be checked, extended, and – per the five predictions of §11 – proven wrong. A synthesis paper of this kind does not close a question so much as hand two literatures a shared vocabulary and a shared dataset requirement, and step back to see whether biologists' and economists' answers, once forced onto the same equation, agree.

A Proofs

A.1 The Price identity (Identity 1)

Consider a fixed set of continuing units $i = 1, \ldots, n$ with time-t shares $\sigma_i \geq 0$, $\sum_i \sigma_i = 1$, characters z_i, time-$(t{+}1)$ shares σ_i' and characters $z_i' = z_i + \Delta z_i$. Define fitness $w_i = \sigma_i'/\sigma_i$ and $\bar{w} = \sum_i \sigma_i w_i = \sum_i \sigma_i' = 1$ (within a closed set of continuers, mean fitness is unity by construction; the general case with entry and exit is treated in Appendix B, where $\bar{w}$ need not be one and is carried explicitly). Then

$$\begin{aligned}\Delta\bar{z} &= \sum_i \sigma_i' z_i' - \sum_i \sigma_i z_i = \sum_i \sigma_i w_i (z_i + \Delta z_i) - \sum_i \sigma_i z_i \\ &= \underbrace{\sum_i \sigma_i (w_i - \bar{w})\, z_i}_{=\,\mathrm{Cov}_s(w,z)} \;+\; \underbrace{\sum_i \sigma_i w_i \Delta z_i}_{=\,\mathbb{E}_s[w\,\Delta z]},\end{aligned}$$

using $\bar{w} = 1$. Dividing by $\bar{w}$ in the general normalization gives Identity 1 in the form of equation (1). No assumption about behavior, equilibrium, rationality, or the origin of the w_i has been made; the identity is bookkeeping, exact in every period, for any partition of the economy. □

A.2 The economic Fisher theorem (Proposition 2)

Let shares evolve in continuous time by the replicator equation $\dot{s}_i = s_i\,(f_i - \bar{f})$ with $\bar{f} = \sum_j s_j f_j$, and let the selection gradient be linear in the character, $f_i = \beta z_i$, with z_i momentarily frozen (the transmission channel switched off). Then

$$\frac{d}{dt}\,\bar{z} \;=\; \sum_i \dot{s}_i z_i \;=\; \sum_i s_i (f_i - \bar{f})\, z_i \;=\; \mathrm{Cov}_s(f, z) \;=\; \beta\ \mathrm{Var}_s(z) \;\geq\; 0,$$

which is equation (2). In discrete time with growth factors $w_i \propto 1 + \beta z_i \Delta t$, the same computation yields $\Delta\bar{z} = \beta\,\mathrm{Var}_s(z)\,\Delta t + O\big((\beta\Delta t)^2\big)$, so the statement holds to first order in the period length. When transmission is switched back on, the total change is the sum of this selection component and $\mathbb{E}_s[w\Delta z]/\bar{w}$, by Identity 1. □

A.3 The drift null (Proposition 4)

Let $w_i = 1 + \varepsilon_i$ with ε_i independent draws from a distribution with mean 0 and variance σ^2, independent of z_i (neutrality). Write the selection numerator

$$S \;=\; \sum_i \sigma_i\,(w_i - \bar{w})(z_i - \bar{z}), \qquad \bar{w} = 1 + \textstyle\sum_i \sigma_i \varepsilon_i.$$

Expanding $w_i - \bar{w} = \varepsilon_i - \sum_j \sigma_j \varepsilon_j$ and using $\sum_i \sigma_i (z_i - \bar{z}) = 0$,

$$S \;=\; \sum_i \sigma_i\,\varepsilon_i\,(z_i - \bar{z}) \;-\; \Big(\sum_j \sigma_j \varepsilon_j\Big) \underbrace{\sum_i \sigma_i (z_i - \bar{z})}_{=\,0} \;=\; \sum_i \sigma_i\,\varepsilon_i\,(z_i - \bar{z}).$$

The numerator is therefore *exactly* a share-and-deviation-weighted sum of the neutral shocks. Independence and $\mathbb{E}[\varepsilon_i] = 0$ give $\mathbb{E}[S] = 0$ and

$$\mathrm{Var}(S) \;=\; \sigma^2 \sum_i \sigma_i^2 \,(z_i - \bar{z})^2,$$

which is equation (3). The reported selection term is $\mathrm{Sel} = S/\bar{w}$; since $\bar{w} = 1 + O_p\big(\sigma\sqrt{\mathrm{HHI}}\big)$, the ratio introduces a bias of order $\sigma^2\mathrm{HHI}$, second-order relative to the fluctuation scale $\sigma\sqrt{\mathrm{HHI}}$, which is why the proposition is stated "to first order." Defining the dispersion-weighted concentration $D_z = \sum_i \sigma_i^2 (z_i - \bar{z})^2 \big/ \sum_i \sigma_i (z_i - \bar{z})^2$, the standard deviation of the neutral selection term is $\sigma\sqrt{D_z\ \mathrm{Var}_s(z)}$, and when the squared deviations $(z_i - \bar{z})^2$ are uncorrelated with the shares, $\mathbb{E}[D_z] \approx \sum_i \sigma_i^2 = \mathrm{HHI}$, giving the corollary $N_\mathrm{e} = 1/\mathrm{HHI}$. The simulation in Appendix C confirms the formula's accuracy at realistic parameter values (predicted versus simulated standard deviations agree to the third decimal place in both test industries). □

A.4 Multi-level recursion (Proposition 3)

Partition units into groups g with group shares $S_g = \sum_{i \in g} s_i$, within-group shares $\sigma_{i|g} = s_i / S_g$, group characters $z_g = \sum_{i \in g} \sigma_{i|g} z_i$, and group fitnesses $W_g = S_g'/S_g$. Applying Identity 1 at the group level,

$$\Delta Z \;=\; \frac{\mathrm{Cov}_G(W_g, z_g)}{\bar{W}} \;+\; \frac{\mathbb{E}_G\big[W_g\, \Delta z_g\big]}{\bar{W}},$$

and each Δz_g inside the transmission term is itself a within-group Price identity, $\Delta z_g = \mathrm{Cov}_{\sigma|g}(w, z)/\bar{w}_g + \mathbb{E}_{\sigma|g}[w \Delta z]/\bar{w}_g$. Substituting yields the two-level decomposition into between-group selection, within-group selection, and within-unit transmission; iterating over deeper nestings (plants in firms in industries in countries) is immediate. The algebra is standard in the multi-level selection literature; the economic reading is developed in Section 7. □

B The decomposition bridge

This appendix proves Proposition 1 by exhibiting the exact algebraic relations between Identity 1 and the three canonical decompositions. Throughout, C denotes continuers, X exiters (active at t only), E entrants (active at $t{+}1$ only); σ_i, σ_i' are shares normalized *within continuers*; φ_i is the unit's (log) productivity; $\Phi_C = \sum_{i \in C} \sigma_i \varphi_i$ the continuer aggregate; and $\omega_i = \sigma_i'/\sigma_i$ with $\bar{\omega} = \sum_i \sigma_i \omega_i = 1$.

B.1 Price and the FHK split of the continuer term

The Foster–Haltiwanger–Krizan decomposition of the continuer contribution is

$$\Delta\Phi_C = \underbrace{\sum_i \sigma_i \,\Delta\varphi_i}_{\text{within}} + \underbrace{\sum_i \Delta\sigma_i \,\big(\varphi_i - \Phi_C\big)}_{\text{between}} + \underbrace{\sum_i \Delta\sigma_i \,\Delta\varphi_i}_{\text{cross}},$$

with $\Delta\sigma_i = \sigma_i' - \sigma_i$ and all deviations taken from the initial aggregate. The Price split of the same object is $\Delta\Phi_C = \mathrm{Sel} + \mathrm{Trans}$ with $\mathrm{Sel} = \mathrm{Cov}_s(\omega, \varphi)$ and $\mathrm{Trans} = \mathbb{E}_s[\omega\, \Delta\varphi]$. Direct computation gives

$$\text{Sel} = \sum_i \sigma_i(\omega_i - 1)(\varphi_i - \Phi_C) = \sum_i \Delta\sigma_i\,(\varphi_i - \Phi_C) = \text{between},$$

$$\text{Trans} = \sum_i \sigma_i\,\omega_i\,\Delta\varphi_i = \sum_i \sigma_i'\,\Delta\varphi_i = \sum_i \sigma_i\Delta\varphi_i + \sum_i \Delta\sigma_i\Delta\varphi_i = \text{within} + \text{cross}.$$

Both relations are exact. The selection flow of the Price identity *is* the FHK between term, and the Price transmission term is within-unit improvement weighted by *realized* (end-of-period) shares. The long-debated "cross term" of the decomposition literature is thereby identified: it is precisely the difference between weighting within-unit change by initial shares and weighting it by realized fitness. Price accounting, derived rather than posited, places improvement on realized weights and the cross term dissolves into transmission — resolving an ambiguity that the decomposition literature has treated as a matter of taste.

B.2 Price and the Olley–Pakes covariance: flow versus stock

The cross-sectional Olley–Pakes decomposition at time t is $\Phi_C(t) = \bar{\varphi}_u(t) + \text{cov}_{OP}(t)$, where $\bar{\varphi}_u = \frac{1}{n}\sum_i \varphi_i$ is the unweighted mean and $\text{cov}_{OP} = \sum_i \left(\sigma_i - \frac{1}{n}\right)\left(\varphi_i - \bar{\varphi}_u\right) = \Phi_C - \bar{\varphi}_u$. Differencing and substituting the Price split of $\Delta\Phi_C$:

$$\Delta\,\text{cov}_{OP} = \Delta\Phi_C - \Delta\bar{\varphi}_u = \text{Sel} \;+\; \underbrace{\sum_i \left(\sigma_i' - \tfrac{1}{n}\right)\Delta\varphi_i}_{\text{size-biased improvement}}\;.$$

The change in the OP covariance therefore equals the period's selection flow *plus* the covariance between end-of-period shares and within-unit improvement. The OP term is a *stock*: the accumulated sediment of past selection flows and size-biased improvements. This is why cross-country OP levels (large and positive in the United States, smaller in Western Europe, near zero in early-transition Eastern Europe) read naturally as the integrated history of how freely selection has been allowed to run — and why a single period's selection flow and the covariance stock must never be conflated, as informal readings of "the reallocation term" sometimes do.

B.3 Price with entry and exit: the Melitz–Polanec form

Let S_C, S_X be the aggregate shares of continuers and exiters at t ($S_C + S_X = 1$) and S_C', S_E' those of continuers and entrants at $t{+}1$ ($S_C' + S_E' = 1$), with group aggregates $\Phi_C, \Phi_X, \Phi_C', \Phi_E'$. The aggregate change is

$$\begin{aligned}\Delta\Phi &= \left(S_C'\Phi_C' + S_E'\Phi_E'\right) - \left(S_C\Phi_C + S_X\Phi_X\right)\\ &= \underbrace{\left(\Phi_C' - \Phi_C\right)}_{\text{continuers: Sel+Trans}} + \underbrace{S_E'\left(\Phi_E' - \Phi_C'\right)}_{\text{entry}} + \underbrace{S_X\left(\Phi_C - \Phi_X\right)}_{\text{exit}},\end{aligned}$$

using $S_C' = 1 - S_E'$ and $S_C = 1 - S_X$. This is exactly the dynamic Olley–Pakes decomposition of Melitz and Polanec, with their continuer term $\Delta\bar{\varphi}_u + \Delta\,\text{cov}_{OP}$ regrouped by B.2 into the Price flow terms. In the generalized-Price language of Kerr and Godfrey-Smith, entrants and exiters are migration classes with reference points Φ_C' and Φ_C: entry contributes exactly when entrants differ from the continuers they join, exit contributes exactly when exiters differ from the continuers they leave. Every term of the empirical decomposition toolkit is thereby an instance of selection accounting; Proposition 1 follows. □

C Simulation details

The drift-null simulation of Section 8 (Figure 3) is a direct Monte Carlo implementation of the neutral benchmark of Proposition 4, with parameters chosen to sit inside the measured empirical ranges of Section 6 rather than to flatter the result.

Design. An industry is a fixed set of N firms with market shares drawn from a rank–size rule $s_r \propto r^{-a}$ and log productivities drawn once from $z_i \sim \mathcal{N}(0, 0.25^2)$; the implied 90/10 productivity ratio of about 1.9 matches the canonical United States within-industry dispersion. Each replicate applies one period of neutral share growth $w_i = 1 + \varepsilon_i$ with ε_i drawn independently from a Laplace distribution with standard deviation 0.20 — fat-tailed, per the measured distribution of firm growth rates — strictly independent of z_i, and computes the realized selection term $\text{Sel} = \sum_i s_i(w_i - \bar{w})(z_i - \bar{z})/\bar{w}$. Two industries are simulated, 20,000 replicates each, fixed seed 20260829:

	N	a	HHI	$N_\text{e} = 1/\text{HHI}$	$\text{Var}_s(z)$
competitive	400	0.35	0.0031	≈ 321	0.055
concentrated	40	1.06	0.1006	≈ 10	0.031

The concentrated industry's tail exponent $a = 1.06$ is the measured Zipf slope of the United States firm-size distribution.

Results. The analytic prediction $\text{SD}(\text{Sel}) = \sigma\sqrt{\sum_i s_i^2(z_i - \bar{z})^2}$ of equation (3) gives 0.00245 for the competitive industry and 0.00793 for the concentrated one; the simulated values are 0.00244 and 0.00799 — agreement to the third decimal place, confirming that the first-order formula is effectively exact at these parameter values. Simulated means are +0.00001 and −0.00013, consistent with zero. The operational headline: a realized "selection effect" exceeding half a log point per period occurs in 49.7 percent of neutral replicates in the concentrated industry, against 4.0 percent in the competitive one. Note also that the ratio of simulated standard deviations (3.3) is smaller than $\sqrt{\text{HHI}_\text{conc}/\text{HHI}_\text{comp}} \approx 5.7$: the two industries differ in $\text{Var}_s(z)$ and in the correlation between shares and deviations, which is why the exact weight $\sum_i s_i^2(z_i - \bar{z})^2$, not the bare Herfindahl, is the object the test statistic of Section 8 uses; the HHI approximation is a reading aid, valid when productivity deviations are size-independent.

Reproducibility. The simulation is a fifty-line script using only a seeded pseudorandom generator; all reported numbers, and the histogram coordinates of Figure 3, are deterministic outputs of seed 20260829.

D Data provenance

Every empirical magnitude quoted in this paper traces to a published, verified source; none is this paper's own estimate. The headline numbers and their provenance:

Magnitude	Value as stated	Source
Within-industry 90/10 TFP ratio, US manufacturing	1.92 (0.651 log points)	[89]
90/10 TFP dispersion, China and India	exceeding 5:1	[49]
Plant TFP persistence (annual AR(1))	0.6–0.8	[89]
OP covariance, US manufacturing	≈ 50 log points	[16]
OP covariance, Western Europe	20–30 log points	[16]
OP covariance, early-transition CEE	near zero or negative	[16]
Net entry share of ten-year US mfg. MFP growth	25–26 percent	[32]
Within-plant share of the same	50–65 percent	[32]
OP covariance, US telecom equipment, 1974→1987	0.01 → 0.32	[69]
Reallocation share of US TFP growth, 1997–2015	about one half	[14]
TFP cost of markup dispersion, US	≈ 15 percent	[14]
Counterfactual TFP gain from US-level allocation	30–50% (China), 40–60% (India)	[49]
US startup rate, late 1980s → 2004–06 → recession	12.0% → 10.6% → below 8%	[24]
US job creation rate, late 1980s → 2004–06	18.9% → 15.8%	[24]
US firm-size Zipf exponent	≈ 1.06	[12]
Firm growth-rate distribution	Laplace; size–variance exponent ≈ −1/6	[86]

Where a source states a range, the range is quoted; where this paper's figures plot a single bar for a stated range (Figure 2), the midpoint is plotted and the caption records the range. The simulation of Appendix C is the paper's own construction and is labelled as such wherever its numbers appear.

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