

# A Neutron Star Is a Redshift Standard

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**Abstract.** NICER’s simultaneous mass–radius measurements have quietly turned several neutron-star surfaces into calculable redshift standards: through  $z_s = (1 - 2GM/R_{\text{NS}}c^2)^{-1/2} - 1$  each pair fixes a surface redshift to a few percent *with no spectral input*, from  $z_s \simeq 0.40$  for the massive pulsar PSR J0740+6620 to the tightest value,  $z_s = 0.257 \pm 0.026$ , for PSR J0437–4715. That inverts forty years of practice. Any credibly identified surface line of known rest energy then tests the equivalence principle for matter at supranuclear density — seven to eight orders of magnitude beyond terrestrial potentials, in the one regime where a strong-sector violation should live. We build the bound algebra that splits an equivalence-principle anomaly  $\eta_{\text{NS}}$  from equation-of-state uncertainty (the equation of state drops out because  $z_s$  comes from NICER, not from the line), audit the checkered history of claimed lines against it — the retracted-in-practice EXO 0748–676 identification above all — and quantify the rotational-broadening wall,  $\Delta E/E \simeq v_{\text{eq}}/c \approx 1.3 \times 10^{-1} (\nu/500 \text{ Hz})(R_{\text{NS}}/12 \text{ km})$ , an order of magnitude larger than the figure usually quoted and the reason narrow surface lines have been absent for a generation: at the measured 552 Hz spin of EXO 0748–676 it smears a line by  $\approx 14\%$ . The sharpest result is a limit, not a promise. Today the bound is set by  $z_s$ , not by the line: at NICER’s  $\sim 10\%$  radius precision the reach on the dilaton couplings floors near  $\beta_X \lesssim 0.09$  per unit coupling however sharp the line, so a Resolve-class ( $\Delta E/E \approx 7.5 \times 10^{-4}$ ) or NewAthena-class ( $5.7 \times 10^{-4}$ ) line is wasted until the standard sharpens — driving  $z_s$  to 3% and 1% moves the floor to  $\sim 0.027$  and  $\sim 0.010$ . The observing case is therefore a mass–radius program as much as a spectroscopy one. A months-old, still-tentative redshifted line in 4U 1820–30 (Iaria et al. 2026,  $1 + z = 1.72$ ) shows the stakes exactly: it uses the line itself to infer both redshift and compactness — the inversion this paper retires — for want of an independent standard. Against MICROSCOPE’s terrestrial  $\eta \sim 10^{-15}$  the neutron star is not a competitor on one ladder but a complementary axis, constraining a different combination of couplings at a potential  $10^7$ – $10^8$  times deeper.

**Keywords:** neutron-star redshift; NICER mass–radius; equivalence principle; strong-field gravity; surface spectral lines; X-ray calorimeter spectroscopy; XRISM.

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## 1 A neutron-star surface has become a redshift standard, and that changes what a line is worth

For forty years the hunt for a spectral line from a neutron-star surface has been, at bottom, a hunt for a redshift. A transition of known laboratory rest energy  $E_0$  observed at  $E_{\text{obs}}$  returns  $1 + z = E_0/E_{\text{obs}}$ , and a surface redshift returns the compactness  $u = GM_{\text{NS}}/R_{\text{NS}}c^2$ ; from the earliest 1984 Tenma-era candidate in 4U 1636–53, noted in the later reviews [71], through the Ginga detection reported in X 1608–52 [52], to the influential  $z = 0.35$  absorption features that Cottam et al. [21] extracted from 28 stacked bursts of EXO 0748–676, the line was wanted because it weighed the star. That program has a clean logic and a poor record. The EXO 0748–676 identification was not reproduced in a deeper 2003 XMM-Newton exposure by the same team [22], and a 552 Hz burst os-

cillation subsequently fixed the star’s spin [32] at a value that Doppler-broadens any surface line far past the narrow widths originally claimed [47].

We argue the arithmetic now runs the other way, and that the reversal is what makes the line valuable. NICER measures  $M_{\text{NS}}$  and  $R_{\text{NS}}$  by modelling the rotating hot-spot waveform of a millisecond pulsar, with no spectral-line input whatsoever [33, 49, 64]; for PSR J0437–4715 the pair is  $M_{\text{NS}} = 1.418 \pm 0.037 M_{\odot}$  and  $R_{\text{NS}} = 11.36^{+0.95}_{-0.63} \text{ km}$  [18]. Passed through the Schwarzschild exterior relation, that pair fixes the surface redshift  $z_s$  to about ten percent *before a single photon of any line is counted* (Appendix C). Once  $z_s$  is known independently, a credibly identified line of known rest energy stops being a way to weigh the star and becomes a way to test the physics of the redshift itself. The step is exact: the same metric that carries the hot-spot photons carries the line photons, so a discrepancy between a line’s redshift and NICER’s  $z_s$  is not an equation-of-state ambiguity but a statement about how matter at supranuclear den-

sity falls. The emitters sit in a QCD-dominated gravitational binding environment at  $\Phi_s/c^2 \sim 0.2$ , seven to eight orders of magnitude deeper than any terrestrial equivalence-principle experiment reaches (Appendix C), and in the one regime where a strong-sector violation of the equivalence principle is expected to live [23, 24].

The live test case arrived while this paper was being written, and it demonstrates the point by doing the opposite. In NICER data on 4U 1820–30, taken hours after a carbon superburst, Iaria et al. [37] report an absorption feature at  $3.8 \pm 0.05$  keV, interpreted as a gravitationally redshifted iron line at  $1+z = 1.72 \pm 0.05$  — an implied compactness  $u \sim 0.33$ , mass  $1.8\text{--}2.3 M_\odot$  and radius  $8.3\text{--}11$  km, at  $2.5\text{--}8\sigma$  and, on the authors’ own statement, still tentative. The construction is exactly the historical inversion: the line supplies *both* the redshift *and* the compactness, because no independent NICER pulse-profile mass–radius measurement of 4U 1820–30 exists to cross-check against, and the authors make no equivalence-principle claim. It is the closest real instance to our proposal and the clearest illustration of why the order of operations is the whole content: fix  $z_s$  first and independently, then let the line be the second, physics-testing probe.

No prior work performs this specific inversion. The nearest precedents are honest to name. DeDeo and Psaltis [26] already proposed a measured surface-line redshift as a strong-field gravity test, but in 2003 there was no equation-of-state-insensitive redshift standard to compare against and the test was not cast in a declared  $\beta$ -coupling model class. Tuna et al. [76] constrain scalar-tensor gravity from NICER mass–radius posteriors *alone*, using the compactness distribution itself as the observable, with no spectral line. Bambi [4] tests a varying fine-structure constant from a spectral line near a compact object, but the object is a black hole and the line is an accretion-disk K $\alpha$  fluorescence line with no independent redshift standard to separate the anomaly from the disk geometry. The structural precedent in a weak field is the white-dwarf lineage: Sirius B’s gravitational redshift, measured to  $89 \pm 16$  km/s by Greenstein et al. [35] and refined to  $80.65 \pm 0.77$  km/s by Joyce et al. [41], is cross-checked against an independent dynamical mass and agrees with degenerate mass–radius theory to 2.5%, and the test now generalises across thousands of white dwarfs [15] — but at  $\Phi/c^2 \sim 10^{-4}$ , more than three orders of magnitude shallower than a neutron-star surface.

This paper makes five contributions, each stated at its own strength. First, the standard-candle inversion (ours): NICER’s photospheric-radius-independent  $z_s$  as the fixed standard and an independently identified line as the probe, with the error budget derived and equation-of-state uncertainty split off from equivalence-principle violation (Section 2). Second, the bound algebra (ours, in a declared model class): a fractional line-versus-standard discrepancy maps onto the  $\beta$ -couplings at extreme binding through the dilaton parametrisation inher-

ited from Corbeil [19] and the  $A^{-1/3}$  composition law of Damour and Donoghue [23] (Section 3). Third, the audit (ours as compilation): every claimed or candidate surface feature, measured against the requirements the bound imposes, with the EXO 0748–676 story told in full and the Iaria feature entered as its newest row (Section 4). Fourth, the rotational-broadening wall quantified with a corrected coefficient and its evasions ranked against *both* spin and magnetic field (ours as computation; Section 5). Fifth, the observing case (structured): the reach is limited by the standard, not the line, so the programme that delivers the first clean bound is a mass–radius programme as much as a spectroscopy one (Section 6). The result belongs to the crossed-fields series [20], where a quantum-relativistic observable is read against the nuclear sector that sets its scale.

## 2 NICER fixes the redshift before any photon of the line is counted

The measurement that makes the standard possible does not use a line at all. NICER records the soft-X-ray arrival times of a millisecond pulsar’s rotating hot spots to better than 100 ns across a 0.2–12 keV band, with a peak effective area near 1900 cm<sup>2</sup> at 1.5 keV [33]; Bayesian pulse-profile modelling then infers the mass, the equatorial radius and the emitting geometry from the shape of that waveform [11, 64]. Gravitational light-bending, Doppler boosting and the rotational modulation of the visible spot area all enter the waveform, so the fit returns  $M_{\text{NS}}$  and  $R_{\text{NS}}$  together — and it does so from timing and continuum photometry, never from the energy of a transition. That is the property the standard needs: the redshift it yields is independent of any assumption about surface composition or line formation.

What the waveform fit uses, and what it pointedly does not, is worth being concrete about. The observables are the energy-resolved pulse shape and its harmonic content; the fitted parameters are the mass, the equatorial radius, the observer inclination and the number, location and temperature of the emitting hot regions. PSR J0030+0451 rotates at 205.53 Hz [49], fast enough that relativistic beaming and the light-travel-time asymmetry across the visible spot imprint compactness information on the pulse, which is why the method separates  $M_{\text{NS}}$  from  $R_{\text{NS}}$  rather than fixing only their ratio. The inference codes have been cross-verified against synthetic waveforms specifically to catalogue which systematics enter and how precision scales with spin and spot size [12], on the data pedigree assembled in the survey paper [11]; the few-percent radius precision the pipeline targets was the stated design goal of the next-generation X-ray-timing programme [79]. None of this machinery reads the energy of a transition, and that omission is

exactly what lets the resulting  $z_s$  stand as a standard a line can be laid against.

**Proposition 1** (The standard; exact given inputs). *For a slowly-compact, spherical exterior the surface redshift is*

$$\begin{aligned} z_s &= \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}}c^2}\right)^{-1/2} - 1 \\ &= (1 - 2u)^{-1/2} - 1, \quad u = \frac{GM_{\text{NS}}}{R_{\text{NS}}c^2}. \end{aligned} \quad (1)$$

*A NICER pair ( $M_{\text{NS}}, R_{\text{NS}}$ ) with published credible intervals maps to  $z_s$  with propagated uncertainty, in the few-percent to twenty-percent class. A line identification tests physics only when  $z$  is known independently; NICER supplies exactly that, and the historical practice — assume a line identification, infer  $z$ , then infer the equation of state — is the inverse the paper retires for equivalence-principle purposes.*

Equation (1) is the Schwarzschild result, and using it for a rotating star is a structured approximation rather than an exact one: at NICER’s target spin rates (J0030 rotates at 205.53 Hz; 49) frame-dragging and oblateness perturb the exterior metric, and the honest treatment carries those corrections in the error budget (Appendix A) rather than in the headline number. What the approximation buys is transparency:  $z_s$  depends on  $M_{\text{NS}}$  and  $R_{\text{NS}}$  only through the compactness  $u$ , so the whole of the input uncertainty is the uncertainty in a single ratio.

That single ratio is where the honesty of the standard lives, and its error budget has a shape worth stating before the reach argument depends on it. The redshift’s fractional error tracks the radius’s, for two compounding reasons. First, the radio-timing mass is the better-measured member of the pair — Shapiro-delay timing fixes PSR J0740+6620’s mass at  $2.08 \pm 0.07 M_{\odot}$  [30] while its radius is known only to about ten percent — so the radius dominates  $\sigma_u/u$ . Second, the metric factor  $(1 - 2u)^{-3/2}$  amplifies a given fractional compactness error more strongly at higher compactness (Appendix A). The two effects pull the same way: the most compact standard, PSR J0740+6620, carries the *largest* fractional redshift error ( $\sim 15$ – $16\%$ ; Appendix C) despite anchoring the deepest potential, while the less compact but exquisitely timed PSR J0437–4715 carries the smallest,  $\sim 10\%$ . A redshift standard is not automatically better for being deeper; it is better for being well measured, and in the current sample depth and precision live in different sources.

The six mapped standards are shown in Fig. 1, and the pattern in them is the argument of this section. The nearest and brightest rotation-powered millisecond pulsar NICER has targeted, PSR J0437–4715, gives the tightest value:  $u = 0.184$ ,  $z_s = 0.257 \pm 0.026$ , a fractional error of 10.3% dominated by the radius term and helped by the informative Parkes radio-timing mass prior

[18] (all mapped values from the seeded Monte-Carlo of Appendix C). It is the precision anchor. The massive pulsar PSR J0740+6620 anchors the opposite, high-redshift end: at  $M_{\text{NS}} \simeq 2.07 M_{\odot}$  its compactness is  $u \simeq 0.25$  and its redshift  $z_s \simeq 0.40$ , the deepest clean potential in the sample. That mass is not itself in dispute — radio Shapiro-delay timing fixes it at  $2.08 \pm 0.07 M_{\odot}$  [30], consistent with the two-solar-mass anchors that preceded it [2, 28] — so the redshift’s uncertainty is almost entirely the radius’s.

The two-pipeline entries in Fig. 1 are deliberate, and they carry the section’s one genuine caveat. For PSR J0030+0451 the same NICER dataset was analysed by two groups with different hot-region parametrisations: Riley et al. [64] report  $R_{\text{NS}} = 12.71^{+0.15}_{-1.19}$  km and  $M_{\text{NS}} = 1.34^{+0.15}_{-0.16} M_{\odot}$ , while Miller et al. [49] report  $R_{\text{NS}} = 13.02^{+1.24}_{-1.06}$  km and  $M_{\text{NS}} = 1.44^{+0.15}_{-0.14} M_{\odot}$ ; these map to  $z_s = 0.205$  and  $z_s = 0.218$  respectively (Appendix C). For PSR J0740+6620 the same split recurs between Riley et al. [65] at  $12.39^{+1.30}_{-0.98}$  km and the Illinois–Maryland analysis of Miller et al. [50], whose larger  $13.7^{+2.6}_{-1.5}$  km maps to a correspondingly lower redshift; the updated-exposure reanalysis of Salmi et al. [67] at  $12.49^{+1.28}_{-0.88}$  km falls between the two, and the pair with a determinate Monte-Carlo redshift, Riley et al. [65] and Salmi et al. [67], map to  $z_s = 0.404$  and  $z_s = 0.397$  (Appendix C). That the added exposure narrows the interval without collapsing the inter-pipeline offset is the point: the spread between paired pipelines is not additional statistical scatter but the model-dependent systematic floor of the method, and it must be propagated as such. A later reanalysis of J0030 makes the point sharper still: Vinciguerra et al. [77] find two competing solution families near  $[1.4 M_{\odot}, 11.5 \text{ km}]$  and  $[1.7 M_{\odot}, 14.5 \text{ km}]$  depending on the assumed hot-region model, so any single-point J0030 redshift would misstate the true uncertainty. We therefore quote a standard as a source *and* a pipeline, never a source alone, and treat the inter-pipeline spread as the irreducible systematic on  $z_s$  until the geometry degeneracy is broken. The newest single-pipeline target, the near-edge-on PSR J0614–3329 at  $R_{\text{NS}} = 10.29^{+1.01}_{-0.86}$  km [48], gives a compact  $z_s = 0.303$  but has no rival-group reanalysis yet, so we carry it as a provisional standard rather than a precision one.

It matters that the redshift and its error come from a channel with independent physical support. NICER’s radii sit inside the same 11–13 km band that gravitational-wave tidal deformability finds by a completely different method (GW170817 gives  $11.9 \pm 1.4$  km for a  $1.4 M_{\odot}$  star under an equation-of-state parametrisation; 1), and the equation-of-state inferences built on the NICER posteriors [46, 61, 66] return a mutually consistent picture of matter at twice nuclear saturation density. Those cross-checks are reassurance, not inputs: the redshift standard deliberately does *not* pass through an equation of state, which is what lets a later line comparison isolate an equivalence-principle anomaly from the

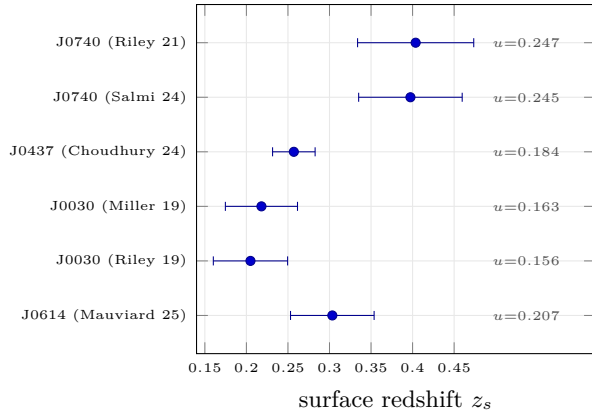


Figure 1: The redshift standards (Proposition 1). Each NICER mass-radius pair maps, through the surface-redshift relation of Eq. (1), to a redshift known to a few percent *with no spectral input*: the massive pulsar PSR J0740+6620 anchors the high-redshift end at  $z_s \simeq 0.40$  [65, 67], while the nearest and best-timed source, PSR J0437–4715, gives the tightest value,  $z_s = 0.257 \pm 0.026$  [18]. Bars are the 68% credible interval propagated from the published  $(M, R_{\text{NS}})$  posteriors by the seeded Monte-Carlo of Appendix C; the compactness  $u = GM/R_{\text{NS}}c^2$  is annotated. Where two pipelines analysed the same data (J0030: 49, 64; J0740: 65, 67) both are shown — the spread between them is the model-dependent systematic floor, not additional statistical error.

nuclear physics. The pre-NICER radius literature, by contrast, spanned 9.9–11.2 km with method-dependent systematics [57] that a next-generation timing instrument was designed to beat to the few-percent level [79]; NICER is that instrument’s first delivery on the promise.

Read together, Fig. 1 is a small catalogue of calculable redshifts: PSR J0740+6620 supplies depth, PSR J0437–4715 supplies precision, and the paired entries supply an honest error bar. None of the six required a spectral line to construct, and that is precisely why a spectral line, laid against one of them, would measure something the star’s structure cannot fake. The next section states exactly what that comparison bounds, and what it does not.

### 3 What a line-versus-standard discrepancy bounds, and what it cannot

The comparison is between two redshifts of the same surface: the one NICER computes from the metric,  $z_s$ , and the one a line reports,  $1 + z_{\text{line}} = E_0/E_{\text{obs}}$ . In general relativity with a universally coupled metric the two are identical, because the line photons and the hot-spot photons climb the same potential. A fractional difference is the observable, and in the dilaton model class this paper

inherits it carries a definite meaning.

**Proposition 2** (The equivalence-principle bound algebra; ours in the declared model). *For emitters of laboratory rest energy  $E_0$  on a surface of independently known  $z_s$ , the measured  $E_{\text{obs}}$  tests*

$$\frac{1 + z_{\text{line}}}{1 + z_s} = 1 + \eta_{\text{NS}}, \quad (2)$$

with  $\eta_{\text{NS}}$  the compact-body redshift anomaly. In the B01 dilaton-class parametrisation of Corbeil [19],

$$\eta_{\text{NS}} \sim \sum_X \Delta K_X \beta_X \frac{\Phi_s}{c^2}, \quad \frac{\Phi_s}{c^2} \sim 0.2, \quad (3)$$

where  $\beta_X$  are the light-dilaton couplings to the Standard-Model sector combinations  $X$  (nuclear binding, the QCD scale, the electron and light-quark masses) and  $\Delta K_X$  are the composition factors that follow the  $A^{-1/3}$  law of Damour and Donoghue [23]. Because  $\Phi_s/c^2$  is seven to eight orders of magnitude above a terrestrial  $\Delta\Phi/c^2$ , a line measured to fractional precision  $10^{-2}$ – $10^{-4}$  bounds the same coupling combinations at  $10^{-2}$ – $(0.2)^{-1}$ -class sensitivity per unit  $\Delta K_X$  and better. The equation of state drops out because  $z_s$  comes from NICER, not from the line; the line-formation physics does not drop out, and each of its systematics must be bounded separately.

The sensitivity in Eq. (3) is set by how well  $\eta_{\text{NS}}$  can be measured, and that is a quadrature of two errors: the line’s fractional precision  $\Delta E/E$  and the standard’s fractional precision  $\sigma_{z_s}/(1 + z_s)$ . Writing  $\sigma_\eta = [\Delta E/E^2 + (\sigma_{z_s}/(1 + z_s))^2]^{1/2}$ , the reach on the couplings per unit composition factor is  $\sigma_\eta(\Phi_s/c^2)^{-1}$  (Appendix A). Evaluated on the precision anchor PSR J0437–4715 ( $z_s = 0.257$ ,  $\Phi_s/c^2 = 0.229$ ; 18), the seeded computation gives the following, and its shape is the section’s central result. A line at  $\Delta E/E = 10^{-2}$  yields  $\sigma_\eta = 2.33 \times 10^{-2}$  and a reach  $\beta_X \lesssim 0.102$  per unit  $\Delta K_X$ ; sharpening the line to  $10^{-3}$  improves  $\sigma_\eta$  only to  $2.11 \times 10^{-2}$  (reach 0.092), and to  $10^{-4}$  only to  $2.10 \times 10^{-2}$  (reach 0.092), all limited by the  $z_s$  error (Appendix C). The crossover is at  $\sigma_\eta \simeq 2.1 \times 10^{-2}$ : once the line is sharper than about  $2 \times 10^{-2}$ , the standard, not the line, sets the bound, and further spectral precision is wasted until NICER’s  $z_s$  improves. This is the quantitative core of the observing case in Section 6, and it inverts the intuition that a sharper line is always worth more.

The theoretical setting for  $\eta_{\text{NS}}$  is the one in which a strong-sector violation is expected to be largest. Scalar-tensor gravity can satisfy every weak-field test and still produce order-unity deviations once compactness crosses a critical value — the spontaneous-scalarization mechanism of Damour and Esposito-Farese [24], already constrained but not excluded by binary-pulsar timing [25] and extended to Gauss-Bonnet couplings for both stars and black holes [68]. A neutron-star surface is where

$\Phi_s/c^2$  and the QCD-dominated binding are both extreme, so Eq. (2) probes exactly the regime the dynamical tests cannot reach through orbital motion. That mapping from a measured  $\eta_{\text{NS}}$  to the couplings is structured, not exact: it holds within the declared B01 class and would be reparametrised in another, which is why we quote reaches “per unit  $\Delta K_X$ ” rather than as model-independent numbers.

Two facts about the bound’s meaning keep it honest. The equivalence principle is, on every weak-field test, extraordinarily well supported: Eötvös-type composition experiments, local-Lorentz-invariance tests and clock comparisons all agree with general relativity, and the Hulse–Taylor binary’s gravitational-wave damping matches the quadrupole prediction to better than half a percent [82]. A neutron-star surface line cannot compete with those on precision, and does not try to; it moves the test to a potential and a matter sector they cannot reach, which is exactly why a comparatively coarse bound there carries information a sharper terrestrial one does not. The second fact is structural: the dominant phenomenology of the light dilaton reduces, in the parametrisation of Damour and Donoghue [23], to essentially two coupling combinations, so one  $\eta_{\text{NS}}$  measurement constrains a definite low-dimensional slice rather than an open-ended family, which is what makes the per- $\Delta K_X$  reaches of Eq. (11) comparable from source to source. The strong-field pulsar-timing programme [81] probes the same theories through orbital dynamics; Section 6 argues that the spectroscopic channel is complementary to it rather than a substitute.

What the equation of state gives up, the line-formation physics claims back, and the discipline of Proposition 2 is to bound every term of it with a number. Three systematics dominate. The first is the intrinsic, non-rotational line profile: the resonant hydrogenic iron features modelled for a burst photosphere by Chang et al. [16] carry Stark broadening, resonant scattering and NLTE level populations, with predicted rest-frame equivalent widths of  $\sim 15\text{--}20$  eV for Fe Ly $\alpha$  and an implied iron column of  $1\text{--}3 \times 10^{20} \text{ cm}^{-2}$  — itself three-to-ten times larger than models comfortably supply, which is a systematic on whether the line exists at the claimed depth at all. The second is the magnetic shift, and it sets the paper’s operational meaning of “low  $B$ ”. Magnetic forces dominate Coulomb forces once  $B \gg 2.35 \times 10^9 \text{ G}$  [45], and any field-linked displacement of a line scales, in the cyclotron limit, as  $E_{\text{cyc}} \approx 11.6 (B/10^{12} \text{ G}) \text{ keV}$  [69]. For a magnetic contribution to sit below a  $\Delta E/E \sim 10^{-3}$  line comparison on a few-keV transition, the field must lie far below  $10^{12} \text{ G}$  — the light-element, partially-ionised surface regime of Lai [45], not the accreting-pulsar regime; H-like iron on a genuinely low-field surface is the species that minimises this term. The third is rotational broadening, treated on its own terms in Section 5, and its full general-relativistic form was established for exactly this purpose by Özel and Psaltis [58] and Chang et al. [17].

Two of these systematics can be converted into cross-checks rather than merely bounded. A surface line and a burst oscillation measured from the same star over-constrain the geometry, because the spin that broadens the line is the same spin the oscillation reveals [9]; a joint fit to both pins the rotational term instead of assuming it. The line-formation term has an independent handle too: direct model-atmosphere fits to burst cooling tails already return masses and radii for individual bursters —  $R_{\text{NS}} = 12.4 \pm 0.4 \text{ km}$  and  $M_{\text{NS}} = 1.9 \pm 0.3 M_{\odot}$  for 4U 1702–429, with an acknowledged 1–5% energy-flux systematic from unmodelled spectral features [53], and a stiff-equation-of-state lower limit of 14 km from 4U 1724–307 [72]. Those fits are atmosphere-model-dependent, so they are not redshift standards in their own right; but a source whose atmosphere fit and whose NICER pulse-profile fit agree is a source whose line-formation systematics are demonstrably under control, which is a selection criterion the observing case can apply.

The atmosphere model itself is a fourth, quieter systematic: whether a given surface is a deep gaseous atmosphere, a condensed radiating surface or a thin finite layer changes the predicted profile [56, 59], and NLTE model-atmosphere synthesis is the tool that decides whether a claimed equivalent width is physically achievable at all [62]. One way to sidestep the whole line-formation tangle is to abandon narrow lines for photoionisation edges: burst-ash edges from radius-expansion bursts can exceed 100 eV equivalent width and be, in the authors’ word, “potentially detectable” [80], trading a resonance line’s clean rest energy for an edge’s larger optical depth. That trade is a design choice we return to in the observing case, not a systematic to be bounded here.

The algebra, then, is permissive about the star and strict about the atom. It asks nothing of the equation of state, because NICER already answered that; it asks a great deal of the line — a known rest energy, a low field, a resolved profile, and a rotationally quiet surface — and it converts each of those demands into a number a real source either meets or fails. The next two sections apply exactly that test: first to every line the literature has claimed, then to the rotation that has erased most of them.

## 4 Every claimed surface line, measured against the requirements

**Proposition 3** (The audit; ours as compilation). *No claimed or candidate neutron-star surface feature to date simultaneously meets the requirements Proposition 2 imposes for an equivalence-principle line: an independently known  $z_s$ , a rest energy fixed by atomic physics rather than by the local field, a profile resolved against the rotational wall, and a surface field low enough that mag-*

netic shifts fall below the intended line precision. Table 1 records each claim with its current status and the specific requirement it fails or, in the newest cases, has not yet been tested against.

The central case is EXO 0748–676, and it must be told in full because it is the reason the field is cautious. Cottam et al. [21] stacked 28 thermonuclear bursts observed with the Chandra grating spectrometer and reported narrow absorption features they identified with the Fe XXVI and Fe XXV  $n = 2$ –3 transitions and the O VIII  $n = 1$ –2 transition, all at a common  $z = 0.35$ , consistent with a normal-matter star of  $1.3$ – $2.0 M_{\odot}$ . On its own the result was clean and quantitative. Three later measurements dismantled it, none of them by finding an error in the original spectra. First, a deeper 2003 XMM-Newton exposure by the same authors, with roughly two orders of magnitude more burst counts, did not reproduce the features [22]. Second, a 552 Hz burst oscillation revealed the spin, an order of magnitude faster than the  $\leq 45$  Hz then assumed [32]; at that spin the equatorial velocity broadens any surface line to  $\Delta E/E \approx 0.14$  (Section 5), which Lin et al. [47] showed is quantitatively incompatible with the narrow widths claimed. Third, and predating the spin measurement, a full general-relativistic population calculation had already found that at 300 (600) Hz only 10–20% (5–10%) of neutron stars would show features as deep as the one claimed [17], while the atmosphere modelling implied an iron column three-to-ten times larger than physics supplies [16]. The identification is retired in practice, and it is retired for a reason the standard-candle inversion would have flagged at the outset: EXO 0748–676 has no independent  $z_s$ , so the line was being asked to measure the star and test the physics at once, which it cannot do.

The older claims fail earlier and more simply. The Ginga-era absorption feature reported in X 1608–52 [52] and the 1984 Tenma-era candidate in 4U 1636–53 noted in the reviews [71] predate both the spin measurements and the modern atmosphere models, and neither has an independent redshift standard; they enter the audit as prior, non-confirmed attempts rather than as usable bounds.

A second class of features is real, repeatedly detected, and disqualified by construction rather than by non-detection. Cyclotron resonance features, such as the proton-cyclotron line in the magnetar SGR 1806–20 [38], have a centroid energy set by the local field,  $E_{\text{cyc}} \propto B$ , not by a fixed atomic rest energy, so they carry no clean gravitational redshift and are excluded as equivalence-principle probes on the same ground for every source. The strongly phase-variable absorption features found in the low-field magnetar SGR 0418+5729 [73] and in the isolated neutron star RX J0720.4–3125 [13], surveyed across the class by Borghese et al. [14], are diagnostic in the opposite way: a genuine gravitational-redshift line from a uniform photosphere should not swing in energy with rotational phase, so their phase dependence is pos-

itive evidence for a magnetically or geometrically confined origin — detected, but excluded.

The candidates that remain open are the recent ones, and they are open precisely because they have not yet been tested against the full requirement set. Radius-expansion bursts with photospheric superexpansion can loft heavy-element ashes into an observable layer [39], the physical circumstance under which a line or edge could form at all, and Barra et al. [6] report line detections during such bursts in 4U 1820–303 — a positive claim whose status this audit does not adjudicate. The newest and most instructive row is the feature Iaria et al. [37] report in 4U 1820–30: an absorption line at  $3.8 \pm 0.05$  keV appearing hours after a carbon superburst, of FWHM  $0.25 \pm 0.05$  keV and equivalent width 15–60 eV, at  $2.5$ – $8\sigma$ , interpreted as a redshifted iron line at  $1 + z = 1.72 \pm 0.05$  and hence  $u \sim 0.33$ , mass  $1.8$ – $2.3 M_{\odot}$ , radius 8.3–11 km. Its FWHM already limits its precision to  $\Delta E/E \sim 0.07$ , well above the calorimeter class. More to the point of this paper, the authors derive the redshift and the compactness *from the same line*, because 4U 1820–30 has no independent NICER pulse-profile mass-radius measurement, and they make no equivalence-principle claim. It is therefore not a counterexample to Proposition 3 but its clearest illustration: the one place in the current literature with a redshifted surface line and a compact-star compactness is exactly the place where the two are not independent, which is the degeneracy the inversion exists to break.

The audit’s verdict, then, is a null with a structure. The failures are not random: the historical claims lacked an independent standard, the confirmed features are field-driven, and the freshest candidates are either unresolved against rotation or self-referential in their redshift. Every one of those failure modes is a specific line of Table 1, and every one is addressable by a deliberate choice of source and instrument. The largest and most systematic of the failure modes — rotation — is the subject of the next section.

## 5 Rotation is the wall, and slow rotation buys past it only at the cost of field

The single largest reason narrow surface lines have been absent for forty years is not sensitivity but rotation, and the coefficient matters enough to state carefully.

**Proposition 4** (The wall and the evasions; ours as computation). *Surface rotation Doppler-broadens any line by*

$$\begin{aligned} \Delta E/E &\simeq \frac{v_{\text{eq}}}{c} = \frac{2\pi\nu R_{\text{NS}}}{c} \\ &\approx 1.3 \times 10^{-1} \left( \frac{\nu}{500 \text{ Hz}} \right) \left( \frac{R_{\text{NS}}}{12 \text{ km}} \right), \end{aligned} \quad (4)$$

Table 1: The audit of claimed and candidate neutron-star surface features against the Proposition 2 requirements (an independent  $z_s$ ; a field-independent rest energy; a profile resolved against rotation; a low surface field). “Retired in practice” denotes a claim the field no longer treats as a detection; “excluded by construction” denotes a real feature that cannot serve as an equivalence-principle probe whatever its significance.

| Claim (source, feature)                                              | Status                               | Verdict against the requirements                                                                                                                                                                          |
|----------------------------------------------------------------------|--------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4U 1636–53, Tenma (1984 candidate); noted in [71]                    | Not confirmed; not re-verified here  | No independent $z_s$ ; predates spin and atmosphere modelling; enters only as a prior non-confirmed attempt, no number attributed.                                                                        |
| X 1608–52, Ginga absorption [52]                                     | Not confirmed                        | No independent $z_s$ ; instrument-limited resolution; superseded by the modern line-formation and spin arguments.                                                                                         |
| EXO 0748–676, $z = 0.35$ Fe/O lines [21]                             | Retired in practice                  | Fails all four: no independent $z_s$ ; 552 Hz spin gives $\Delta E/E \approx 0.14$ , incompatible with narrow widths [32, 47]; not reproduced in deeper data [22]; column 3–10 $\times$ over models [16]. |
| SGR 1806–20, proton-cyclotron feature [38]                           | Detected; excluded by construction   | $E_{\text{cyc}} \propto B$ , not a fixed atomic rest energy: no clean gravitational redshift, excluded as an EP probe for every source.                                                                   |
| SGR 0418+5729 and RX J0720.4–3125, phase-variable lines [13, 14, 73] | Detected; excluded by construction   | Centroid swings with rotational phase: evidence for magnetic/geometric confinement, the opposite of a uniform-photosphere EP line.                                                                        |
| Superexpansion burst ashes (edges) [39, 80]                          | Predicted mechanism                  | Supplies the line-forming species; an edge trades a clean rest energy for larger optical depth — a candidate route, not yet a bound.                                                                      |
| 4U 1820–303, PRE-burst line detections [6]                           | Claimed (2025); not adjudicated here | Recent positive claim during superexpansion; requirement test (independent $z_s$ , field, resolution) pending full assessment.                                                                            |
| 4U 1820–30, 3.8 keV line, $1 + z = 1.72$ [37]                        | Tentative (2.5–8 $\sigma$ )          | The historical inversion in a modern instrument: line supplies <i>both</i> $z$ and $u$ ; no independent NICER $z_s$ ; FWHM limits $\Delta E/E \sim 0.07$ ; no EP claim made.                              |

where  $\nu$  is the spin frequency. Equation (4) is the wall that erases narrow lines on ordinary fast bursters. Evasion is possible along four axes, ranked by how much line precision they preserve: (i) slow rotators ( $\nu \lesssim 1$  Hz), smearing  $10^{-5}$  class; (ii) near-pole-on viewing geometries, from the pulse-modelling inclination posteriors; (iii) magnetically confined polar-cap emission — but then a field-dependent shift replaces the rotational one; and (iv) absorption during photospheric-radius-expansion bursts of slow sources. The slow-rotation escape (i) is in tension with a clean gravitational redshift, because the slowest rotators carry the strongest fields.

The corrected coefficient in Eq. (4) is a “we” result, and it is an order of magnitude larger than the percent figure commonly quoted: evaluating  $2\pi\nu R_{\text{NS}}/c$  at  $\nu = 500$  Hz and  $R_{\text{NS}} = 12$  km gives  $\Delta E/E = 0.126$ , not  $10^{-2}$  (Appendix C). The consequence for the historical record is decisive. At the 552 Hz spin measured for EXO 0748–676 [32] the smearing is  $\Delta E/E \approx 0.14$ , so the surface lines there are broadened to roughly a seventh of their own energy; no calorimeter of any resolution recovers a narrow feature from that, and it was the wall, not the instrument, that defeated the claim. Figure 2 plots Eq. (4) against a Resolve-class resolution line at  $\Delta E/E \sim 10^{-3}$  and marks the shortlist sources, and the geometry of the plot is the argument: a source is a candidate only if it sits below the resolution line.

Slow rotation drops a source far below it. The seeded computation gives, at  $R_{\text{NS}} = 12$  km,  $\Delta E/E = 2.0 \times 10^{-4}$  for Her X-1 (spin period 1.24 s; 10),  $3.3 \times 10^{-5}$  for 4U 1626–67 (7.66 s; 10, 74), and below  $10^{-6}$  for the wind-

fed slow rotator Vela X-1 and for the two-minute rotator GX 1+4 (Appendix C). Every one of these clears the rotational wall by orders of magnitude. Two of the four evasion geometries are geometric rather than kinematic and can preserve narrow lines even on faster stars. The first is viewing angle: the full general-relativistic ray-tracing of Bauböck et al. [8] identifies the spin-and-inclination combinations under which a near-pole-on view, where the line of sight sees little of the equatorial velocity field, keeps a line detectable despite rapid rotation. What makes this more than a hope is that the same NICER pulse modelling that fixes  $z_s$  also returns the inclination posterior, so the geometry is measured, not assumed — though it can cut the wrong way: the near-edge-on orientation inferred for PSR J0614–3329 [48] is the worst case for smearing, while the non-antipodal spot geometry of PSR J0437–4715 [18] shows how source-specific the answer is. The second geometric route is transient: superexpansion during a photospheric-radius-expansion burst [39] lofts heavy-element ashes into an observable layer in which a line can form at all, and the burst catalogue that calibrates such bursts as standard candles [44] supplies the target list and flux normalisation. The confined-polar-cap route (iii) is the treacherous one: magnetically confining the emission to a small cap keeps it localised, but the field that confines it displaces the line, and the phase-variable feature in SGR 0418+5729 [73] is the observational demonstration that a confined feature’s energy then swings with rotational phase. All three routes are real; all three are conditional — pole-on orientation is a matter of the source’s luck, superexpansion a matter of catching the right seconds of a rare burst, and con-

finement a bargain that reintroduces the field problem it was meant to avoid.

The difficulty that turns this from a shortlist into a genuine problem is that the slow-rotation escape and the clean-redshift requirement pull in opposite directions, and the pull is quantitative. The slowest rotators are overwhelmingly accreting or isolated magnetic stars. Her X-1 shows a cyclotron line at  $37.44 \pm 0.07$  keV [70], which through  $E_{\text{cyc}} \approx 11.6 (B/10^{12} \text{ G}) \text{ keV}$  [69] implies a surface field near  $3 \times 10^{12} \text{ G}$ ; worse for a redshift test, that line energy rises about 6.5% when the source flux doubles [70], so its centroid is coupled to both field and accretion rate. 4U 1626–67 carries a comparable  $\sim 4 \times 10^{12} \text{ G}$  field and a  $\sim 36$  keV cyclotron line [74]. Magnetars, the extreme slow rotators at 2–12 s spin, sit at  $B \gg 10^{14} \text{ G}$  [42, 55], far into the regime where the field, not gravity, sets every line energy. At the fields that accompany slow rotation in these sources, a  $10^{12} \text{ G}$ -class shift dwarfs a  $10^{-3}$  line comparison by orders of magnitude, so the cyclotron-line sources are excluded as equivalence-principle probes precisely for the property that makes them detectable at all. Slow rotation evades the wall; strong field rebuilds it in a different place.

The target that satisfies both axes is therefore narrow and specific: a slow rotator with a genuinely low surface field, so that the line is neither smeared by spin nor displaced by magnetism. Naming that requirement quantitatively is the real observing-case problem, and it is why the shortlist in Table 2 carries a field column beside the spin column. Most rows fail on field having passed on spin. The one class that plausibly passes both is the central compact objects — the “anti-magnetar” remnants with low inferred dipole fields and reported surface absorption features; the slow CCO 1E 1207.4–5209 sits at  $\Delta E/E \sim 2 \times 10^{-5}$  in the seeded computation (Appendix C) and is the archetype the observing case points to, with the caveat that its detailed parameters are adopted, not independently re-verified here. The honest boundary of Proposition 4 is thus a scarcity statement: the wall is evadable, but the sources that evade it cleanly are rare, and the next section asks whether any real (source, instrument, geometry) triple among them can reach a bound worth having.

## 6 The first clean line is a mass–radius problem before it is a spectroscopy problem

**Proposition 5** (The observing case; structured). *Calorimeter-class spectrometers already resolve a surface line far below the precision the standard can currently exploit. XRISM Resolve delivers  $\sim 4.5 \text{ eV}$  at  $6 \text{ keV}$ ,  $\Delta E/E \approx 7.5 \times 10^{-4}$  [40], and NewAthena X-IFU is specified at  $4 \text{ eV}$  up to  $7 \text{ keV}$ ,  $\Delta E/E \approx 5.7 \times 10^{-4}$  [54] — descoped from the original  $2.5 \text{ eV}$  design [7] but still*

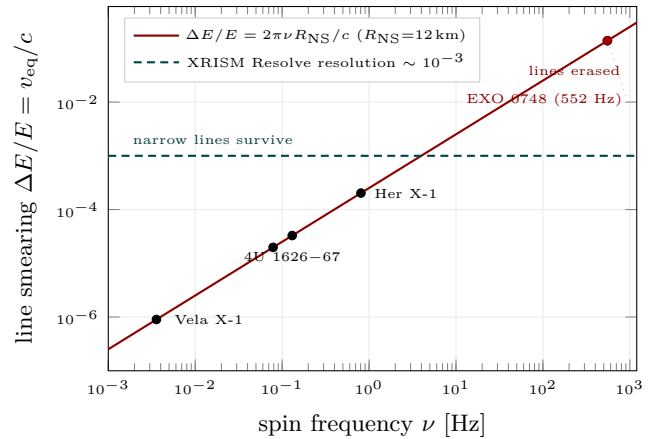


Figure 2: The rotational-broadening wall (Proposition 4). Surface rotation Doppler-broadens any line by  $\Delta E/E \simeq v_{\text{eq}}/c = 2\pi\nu R_{\text{NS}}/c \approx 1.3 \times 10^{-1} (\nu/500 \text{ Hz})(R_{\text{NS}}/12 \text{ km})$  — an order of magnitude larger than the commonly quoted per-cent figure, and the reason narrow surface lines have been absent for forty years. At the 552 Hz spin measured for EXO 0748–676 [32] the smearing is  $\approx 14\%$ , which no 5 eV calorimeter can resolve as a narrow feature; the wall, not sensitivity, is what the historical claims ran into [8, 17]. The evasion is spin: slow accretors such as Her X-1 (1.24 s) and 4U 1626–67 (7.66 s) fall below the Resolve resolution line, where a narrow line survives. Values from the seeded computation of Appendix C.

*calorimeter class. Against the bound of Proposition 2, both over-deliver: at today’s  $\sim 10\%$  NICER radius precision the reach floors at  $\beta_X \lesssim 0.09$  per unit  $\Delta K_X$  however sharp the line, and a line better than  $\Delta E/E \sim 2 \times 10^{-2}$  is wasted until the standard sharpens. Driving  $z_s$  to 3% and 1% moves the floor to  $\sim 0.027$  and  $\sim 0.010$  (Appendix C). The observing case is therefore a mass–radius programme as much as a spectroscopy one.*

This is the paper’s sharpest and most uncomfortable conclusion, and Fig. 3 is its statement. The reach curve is nearly flat across three decades of line precision and moves only when the standard’s error moves; the calorimeter band sits well inside the region where the line has stopped mattering. Spectroscopy is not the bottleneck. The bottleneck is the radius, and the fastest way to a stronger equivalence-principle bound from a neutron-star surface is to measure a better mass and radius for the source that carries the line — to shrink  $\sigma_{z_s}$  on the precision anchor PSR J0437–4715 [18] and its successors, exactly the programme the NICER collaboration is already running for equation-of-state reasons [48, 67]. A line-search proposal that does not also secure an independent mass–radius measurement for its target is proposing half an experiment.

The exposure arithmetic sets which sources are reachable at all. NICER’s own background floor is a me-

Table 2: The evasion shortlist ranked against *both* rotational smearing (Eq. 4) and surface field. Smearing values are at  $R_{\text{NS}} = 12$  km from the seeded computation of Appendix C; a source is a viable equivalence-principle target only if it clears the Resolve-class resolution ( $\Delta E/E \lesssim 10^{-3}$ ) *and* carries a field low enough that magnetic shifts fall below the intended line precision.

| Source         | Spin                   | $\Delta E/E$            | Field / contamination                                                      | Verdict                                                                           |
|----------------|------------------------|-------------------------|----------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| EXO 0748–676   | 552 Hz [32]            | $1.4 \times 10^{-1}$    | Low-field burster, but irrelevant here                                     | Disqualified by the wall: the disapproved $z = 0.35$ source.                      |
| Her X-1        | 1.24 s [10]            | $2.0 \times 10^{-4}$    | $B \sim 3 \times 10^{12}$ G; cyclotron 37.4 keV, flux-coupled [69, 70]     | Clears the wall; excluded on field — line energy set by $B$ and accretion.        |
| 4U 1626–67     | 7.66 s [10, 74]        | $3.3 \times 10^{-5}$    | $B \sim 4 \times 10^{12}$ G; cyclotron $\sim 36$ keV [74]                  | Deep evasion of the wall; excluded on field.                                      |
| GX 1+4         | $\sim 2$ min [34]      | $\sim 2 \times 10^{-6}$ | Accreting magnetic pulsar; flux variable, multi-year off states            | Far below the wall; excluded on field/accretion and observability.                |
| Vela X-1       | $\sim 283$ s (adopted) | $9.0 \times 10^{-7}$    | Wind-fed high-field HMXB [10]                                              | Essentially unbroadened; excluded on field/accretion.                             |
| 1E 1207.4–5209 | slow (CCO; adopted)    | $2.0 \times 10^{-5}$    | Low inferred dipole field (anti-magnetar class); reported surface features | Plausibly clears <i>both</i> axes; the archetype target, parameters adopted here. |

dian 0.87 counts/s over its detectors, with a  $3\sigma$  single-interval detection limit of 1.20 counts/s, or  $3.6 \times 10^{-12}$  erg cm $^{-2}$  s $^{-1}$  for a Crab-like spectrum, and a softer-band limit of  $4.3 \times 10^{-13}$  erg cm $^{-2}$  s $^{-1}$  for a 100 eV blackbody [63]; the bursting-source pool from which candidates are drawn is catalogued in MINBAR at 7083 thermonuclear bursts across 85 confirmed sources [31], with the peak-flux normalisation for the radius-expansion bursts among them set by the standard-candle calibration of Kuulkers et al. [44]. For a calorimeter the operative caution is the opposite of faintness: at the high count rates of a bright burst, Resolve’s resolution degrades through inter-pixel cross-talk and must be recovered with a nearest-neighbour coincidence cut [51], and the original X-IFU design already anticipated degradation to  $\sim 10$  eV at Crab intensities [7]. A slow, low-field target observed during a superexpansion burst therefore has to be bright enough to build line counts yet not so bright that the calorimeter loses the resolution the whole case depends on. Large-area timing missions such as eXTP [29] help on the mass-radius side of the ledger — more pulse-profile statistics — but they are not calorimeters and do not resolve the line; the two instrument classes are complementary, not interchangeable.

The comparison with terrestrial equivalence-principle tests is with them, not against them, and the spec’s “ladder” framing is the wrong picture. MICROSCOPE bounds the weak-equivalence-principle Eötvös parameter at  $\eta = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$  [75], and orbiting hydrogen masers confirm the gravitational redshift to the  $10^{-5}$  level [27, 36]; both live at  $\Delta\Phi/c^2 \sim 10^{-10}$ , seven to eight orders of magnitude shallower than a neutron-star surface at  $\Phi_s/c^2 \sim 0.2\text{--}0.4$  (Appendix C). The surface line does not beat MICROSCOPE’s precision — it cannot, at present  $z_s$  errors — but it constrains a different combination of the dilaton couplings, because the  $A^{-1/3}$  compo-

sition law [23] weights a QCD-dominated neutron- star surface differently from a Ti–Pt laboratory pair. It is a complementary axis at a vastly deeper potential, not a rung above the terrestrial bound. The same complementarity separates it from the dynamical strong-field tests: the differential free-fall bounds from the pulsar triple system,  $\Delta < 2.6 \times 10^{-6}$  [3] and  $(0.5 \pm 1.8) \times 10^{-6}$  [78], and the double-pulsar agreement with general relativity at  $1.3 \times 10^{-4}$  [43], are dynamical, orbital-motion observables; the line test is a spectroscopic one, and the two are not convertible into a single number without the explicit model mapping. The nearest structural precedent remains the white-dwarf redshift cross-checked against an independent mass [41], now scaled up here by more than three orders of magnitude in potential.

The programme belongs to the older effort to test strong-field gravity with electromagnetic observations [60], but with an observable that effort did not have before NICER: a surface line read against a mass-radius-derived redshift standard rather than against itself. That difference bears directly on the burden of proof. The field’s current consensus is that no variation of a fundamental constant has been confirmed anywhere [5], so a single neutron-star line anomaly would be an extraordinary claim, and the inversion is what makes it a disciplined one — it removes the largest loophole, an unknown redshift, before the anomaly is asserted, which is exactly the loophole every historical claim in Section 4 left open.

The verdict is a triple and a boundary. The triple that first delivers a bound worth having is a slow, low-field rotator — a central compact object, or a carefully vetted slow accretor in a quiescent low-field state — observed with XRISM Resolve or NewAthena X-IFU, in a near-pole-on geometry or during a superexpansion burst, and, decisively, backed by an independent NICER-class mass-

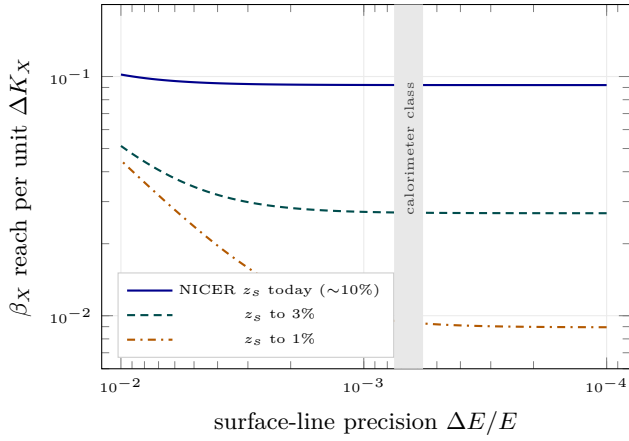


Figure 3: What one surface line buys, and what caps it (Propositions 2 and 5). Taking the tightest standard, PSR J0437–4715 ( $z_s = 0.257$ ,  $\Phi_s/c^2 \simeq 0.23$ ; [18]), the  $\eta_{\text{NS}}$  bound and hence the per-unit- $\Delta K_X$  reach on the dilaton couplings of Damour and Donoghue [23] follows from adding the line precision and the  $z_s$  precision in quadrature. The decisive fact is that  $z_s$ , *not the line*, *is the floor*: at today’s  $\sim 10\%$  NICER radius precision the reach saturates near  $\beta_X \lesssim 0.09$  however sharp the line, so a Resolve-class  $\Delta E/E \approx 7.5 \times 10^{-4}$  [40] or a NewAthena-class  $5.7 \times 10^{-4}$  [54] line is wasted until the standard sharpens — driving  $z_s$  to 3% and 1% moves the floor to  $\sim 0.027$  and  $\sim 0.010$ . The observing case is therefore a mass–radius program as much as a spectroscopy one. The comparison is with, not against, MICROSCOPE’s terrestrial  $\eta = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$  [75]: that bound lives at  $\Delta\Phi/c^2 \sim 10^{-10}$ , seven to eight orders below the neutron-star surface, and constrains a different combination of the couplings — complementary axes, not one ladder. Values from the seeded computation of Appendix C.

radius measurement of that same source. The boundary is honest and specific: the test fails if no slow rotator with a field below the  $10^{12}$  G scale also shows a resolved, field-independent surface line; it fails if the target’s  $z_s$  cannot be driven below the few-percent level that Fig. 3 requires; and it would be killed outright by a demonstration that any candidate line’s energy varies with rotational phase or accretion flux, the signature that disqualified the magnetar features of Section 4. Within those limits, a single confirmed clean line, laid against a NICER redshift standard, would be — more speculatively, but the arithmetic supports it — the deepest-potential equivalence-principle statement available to observation, made in the one regime where a strong-sector violation is expected to live.

## A Derivations: the standard, the bound, and the error budget

This appendix collects the algebra behind Propositions 1, 2 and 4. The numerical evaluations quoted in the text and plotted in the figures are produced by the seeded computation of Appendix C; what follows are the closed forms it evaluates.

### A.1 The standard and its error propagation

For a static, spherical exterior the redshift of a photon emitted at the surface and received at infinity is fixed by the Schwarzschild metric,

$$1 + z_s = (1 - 2u)^{-1/2}, \quad u = \frac{GM_{\text{NS}}}{R_{\text{NS}}c^2}, \quad (5)$$

which is Eq. (1). The redshift depends on  $M_{\text{NS}}$  and  $R_{\text{NS}}$  only through the compactness  $u$ , so the input uncertainty reduces to the uncertainty in one ratio,

$$\frac{\sigma_u}{u} = \left[ \left( \frac{\sigma_M}{M_{\text{NS}}} \right)^2 + \left( \frac{\sigma_R}{R_{\text{NS}}} \right)^2 \right]^{1/2}, \quad (6)$$

under the (structured) assumption that the published mass and radius errors are approximately independent; where a strong  $M$ – $R$  posterior correlation is reported, the seeded computation samples the joint posterior directly rather than using Eq. (6). Differentiating Eq. (5),

$$\frac{dz_s}{du} = (1 - 2u)^{-3/2}, \quad \sigma_{z_s} = (1 - 2u)^{-3/2} \sigma_u. \quad (7)$$

The factor  $(1 - 2u)^{-3/2}$  amplifies the fractional error as  $u$  grows, so a more compact star carries a proportionally larger redshift error for the same  $\sigma_u/u$ ; this, with the fact that NICER radii are less precisely determined than the radio-timing masses, is why the redshift error is dominated by the radius term and why the high-compactness PSR J0740+6620 has a larger fractional  $z_s$  error ( $\sim 15$ – $16\%$ ) than the less compact but better-timed PSR J0437–4715 ( $\sim 10\%$ ; Appendix C). Rotation perturbs Eq. (5) through frame-dragging and oblateness at order  $(\nu/\nu_K)^2$  with  $\nu_K$  the mass-shedding frequency; for the rotation-powered millisecond pulsars used as standards this is a sub-dominant correction carried in the systematic budget, not in the central value.

### A.2 The equivalence-principle bound

Let the line report  $1 + z_{\text{line}} = E_0/E_{\text{obs}}$  from a laboratory rest energy  $E_0$ , and let  $z_s$  be the NICER standard for the same surface. Define the compact-body redshift anomaly by

$$1 + \eta_{\text{NS}} \equiv \frac{1 + z_{\text{line}}}{1 + z_s}, \quad (8)$$

which is Eq. (2); in general relativity with a universally coupled metric  $\eta_{\text{NS}} = 0$  identically, because both redshifts are the same metric factor Eq. (5). In the B01 dilaton class inherited from Corbeel [19], a light scalar couples to the Standard-Model sector combinations  $X$  with strengths  $\beta_X$ , and the surface transition acquires a composition- and potential-dependent shift

$$\eta_{\text{NS}} \simeq \sum_X \Delta K_X \beta_X \frac{\Phi_s}{c^2}, \quad \Delta K_X \propto A^{-1/3}, \quad (9)$$

the composition factors  $\Delta K_X$  following the  $A^{-1/3}$  dependence of Damour and Donoghue [23]. Equation (9) is the structured step: it holds within the declared model class and fixes the physical content of a nonzero  $\eta_{\text{NS}}$ , but its coefficients are reparametrised in a different coupling model, so bounds are quoted per unit  $\Delta K_X$ .

The measurement error on  $\eta_{\text{NS}}$  follows from Eq. (8) by propagating the line and standard errors in quadrature,

$$\sigma_\eta = \left[ \Delta E/E^2 + \left( \frac{\sigma_{z_s}}{1+z_s} \right)^2 \right]^{1/2}, \quad (10)$$

with  $\Delta E/E$  the fractional line precision. The reach on the couplings per unit composition factor is then

$$\beta_X|_{\text{reach}} = \frac{\sigma_\eta}{\Phi_s/c^2}. \quad (11)$$

For the precision anchor PSR J0437–4715 the seeded computation uses  $z_s = 0.257$ ,  $\sigma_{z_s}/(1+z_s) = 2.11 \times 10^{-2}$  and  $\Phi_s/c^2 = 0.229$ . In the line-dominated limit  $\Delta E/E \gg \sigma_{z_s}/(1+z_s)$ , Eq. (10) gives  $\sigma_\eta \rightarrow \Delta E/E$ ; in the standard-dominated limit  $\Delta E/E \ll \sigma_{z_s}/(1+z_s)$  it gives  $\sigma_\eta \rightarrow \sigma_{z_s}/(1+z_s) = 2.11 \times 10^{-2}$ , the floor, and hence  $\beta_X|_{\text{reach}} \rightarrow 0.092$ . The crossover, where the two terms of Eq. (10) are equal, is at  $\Delta E/E \simeq 2.1 \times 10^{-2}$ : a line sharper than this contributes negligibly until  $\sigma_{z_s}$  is reduced. Fixing a Resolve-class  $\Delta E/E = 10^{-3}$  and improving the standard, Eq. (11) gives  $\beta_X|_{\text{reach}} = 9.2 \times 10^{-2}$ ,  $2.7 \times 10^{-2}$  and  $1.0 \times 10^{-2}$  at  $\sigma_{z_s}/(1+z_s) = 10.3\%$ , 3% and 1% respectively (Appendix C).

### A.3 Systematics formulas

*Rotational broadening.* The equatorial rotational velocity is  $v_{\text{eq}} = 2\pi\nu R_{\text{NS}}$ , so the fractional Doppler width of a surface line is

$$\begin{aligned} \Delta E/E &\simeq \frac{v_{\text{eq}}}{c} = \frac{2\pi\nu R_{\text{NS}}}{c} \\ &= 1.26 \times 10^{-1} \left( \frac{\nu}{500 \text{ Hz}} \right) \left( \frac{R_{\text{NS}}}{12 \text{ km}} \right), \end{aligned} \quad (12)$$

the corrected coefficient of Eq. (4): the direct evaluation  $2\pi(500 \text{ Hz})(12 \text{ km})/c = 0.126$  is an order of magnitude larger than the per-cent figure sometimes quoted, and the rounded fiducial  $1.3 \times 10^{-1}$  is used in the text. The

full general-relativistic profile adds oblateness, frame-dragging and Doppler-beaming corrections to Eq. (12) [8, 17, 58]; Eq. (12) is the leading flat-space term and the one that sets the wall.

*Magnetic shift and the meaning of “low  $B$ ”.* A field-linked displacement of a line scales, in the cyclotron limit, as

$$E_{\text{cyc}} \approx 11.6 \left( \frac{B}{10^{12} \text{ G}} \right) \text{ keV} \quad (13)$$

for the electron cyclotron energy [69], with the proton value smaller by  $m_e/m_p$ . Requiring the field-linked fractional shift on a transition of observed energy  $E_{\text{obs}}$  to fall below a target line precision,  $E_{\text{cyc}}/E_{\text{obs}} \lesssim \Delta E/E$ , gives for a few-keV line and  $\Delta E/E \sim 10^{-3}$  a bound  $B \lesssim 10^9 \text{ G}$ -class — far below the accreting-pulsar fields of  $\sim 10^{12} \text{ G}$  and consistent with the light-element, partially-ionised surface regime,  $B \ll 10^{14} \text{ G}$ , of Lai [45]. This is the quantitative content of the “low- $B$ ” requirement in Proposition 2.

*Line-formation shifts.* Stark broadening, resonant scattering and NLTE level populations set the intrinsic (pre-rotation) profile and a pressure-dependent shift; for burst-photosphere hydrogenic iron these give rest-frame equivalent widths of order 15–20 eV and an implied iron column of  $1\text{--}3 \times 10^{20} \text{ cm}^{-2}$  [16], both to be carried as systematics on the existence and centroid of the line independently of the rotational and magnetic terms above. The edge alternative replaces the resonance line with a photoionisation edge of  $\gtrsim 100 \text{ eV}$  equivalent width [80], trading a sharp rest energy for larger optical depth.

## B The shortlist sources and their provenance

Table 3 gives the parameters behind the ranking of Section 5 and Table 2, with a source cited for each value. The smearing  $\Delta E/E$  is computed from Eq. (12) at  $R_{\text{NS}} = 12 \text{ km}$  in the seeded computation of Appendix C; spins, fields and fluxes carry their own literature citations. Two provenance cautions govern the table. First, the spins of the accreting sources are not constants: 4U 1626–67 is in an ongoing spin-down episode [74], and GX 1+4’s pulse period has lengthened by roughly 50% over three decades and anti-correlates with accretion flux [34], so each entry is an epoch value, not a fixed frequency. Second, the parameters of Vela X-1 and of the central compact object 1E 1207.4–5209 are adopted as inputs to the seeded computation rather than independently re-verified in the reference audit; they are flagged as such and should not be quoted from this table as measured values.

The field column is the decisive one for equivalence-principle use, and its provenance is worth stating explicitly. Her X-1’s field is inferred from its  $37.44 \pm 0.07 \text{ keV}$  cyclotron line [70] through the electron cyclotron relation  $E_{\text{cyc}} \approx 11.6 (B/10^{12} \text{ G}) \text{ keV}$  [69], a structured inversion

giving  $B \sim 3 \times 10^{12}$  G; 4U 1626–67’s  $\sim 4 \times 10^{12}$  G comes directly from its  $\sim 36$  keV cyclotron line [74]. Magnetar fields of  $B \gg 10^{14}$  G are catalogued by Olausen and Kaspi [55], updated in the review of Kaspi and Beloborodov [42]. The persistent-accretor spin baselines are from the BATSE monitoring of Bildsten et al. [10]. Exposure context — burst rates and fluences for the bursting sources, and the NICER background floor against which a line must be built — is set by MINBAR [31] and the NICER background model [63], with the caution that MINBAR’s 1996–2012 window pre-dates NICER and current-epoch fluxes must be checked against present monitoring.

## C The seeded computation

The redshift table, the rotational-broadening wall, and the reach curves come from one deterministic script, `sim_nstar.py`, whose source and outputs (`sim_nstar_output.txt/.json`) ship with the paper. The only stochastic step is the propagation of the published NICER mass–radius posteriors into  $z_s$ ; it is seeded (20260919) so the reported percentiles are reproducible to the digit.

### The standards

For each source the script draws  $M$  and  $R_{\text{NS}}$  from two-piece (split) normals matched to the published 68% half-widths,  $4 \times 10^5$  draws each, forms the compactness  $u = GM/R_{\text{NS}}c^2$ , discards the measure-zero unphysical tail  $2u \geq 1$ , and reports the median and 16/84 percentiles of  $z_s = (1 - 2u)^{-1/2} - 1$  (Fig. 1, Proposition 1). The high-redshift anchor is PSR J0740+6620 at  $z_s = 0.404^{+0.070}_{-0.062}$  (Riley pipeline) or  $0.397^{+0.062}_{-0.058}$  (Salmi),  $u \simeq 0.25$ ; the precision anchor is PSR J0437–4715 at  $z_s = 0.257^{+0.026}_{-0.027}$ , the tightest of the set at  $\sim 10\%$  thanks to its 2.6% mass. PSR J0030+0451 gives  $z_s = 0.205$  (Riley) versus 0.218 (Miller) and PSR J0614–3329  $z_s = 0.303$ ; where two pipelines analysed one dataset the gap between them is the model-dependent systematic floor, carried as such and never averaged away.

### The wall

The equatorial Doppler width of a surface line is  $\Delta E/E = v_{\text{eq}}/c = 2\pi\nu R_{\text{NS}}/c$ , which at the fiducial  $\nu = 500$  Hz,  $R_{\text{NS}} = 12$  km is  $1.26 \times 10^{-1}$  — an order of magnitude above the per-cent figure often quoted, and the correct scale of the wall (Fig. 2, Proposition 4). At the measured 552 Hz spin of EXO 0748–676 the smearing is  $1.4 \times 10^{-1}$ ; a narrow line cannot survive it. The evasion is slow rotation: the script tabulates  $2.0 \times 10^{-4}$  for Her X-1 (1.24 s),  $3.3 \times 10^{-5}$  for 4U 1626–67 (7.66 s), and  $9 \times 10^{-7}$  for a wind-fed  $\sim 280$  s rotator — but these are the strongly magnetized sources whose cyclotron lines

disqualify them as clean gravitational probes, the tension Section 5 makes central.

## The reach and the complementarity

Taking J0437–4715 as the reference, the anomaly bound is the quadrature sum  $\sigma_{\eta_{\text{NS}}} = [\delta^2 + (\sigma_{z_s}/(1 + z_s))^2]^{1/2}$  of the fractional line precision  $\delta$  and the absolute  $z_s$  error, and the reach on the dilaton couplings is  $\beta_X \lesssim \sigma_{\eta_{\text{NS}}}/(\Phi_s/c^2)$  per unit  $\Delta K_X$  with  $\Phi_s/c^2 = \ln(1 + z_s) = 0.229$  (Fig. 3, Propositions 2, 5). The decisive output is that the  $z_s$  term,  $2.1 \times 10^{-2}$  today, dominates for any  $\delta < 2 \times 10^{-2}$ : the reach floors at  $\beta_X \simeq 0.09$  however sharp the line, and a Resolve- or NewAthena-class line ( $\delta \approx 6\text{--}8 \times 10^{-4}$ ) is wasted until NICER radii improve. Fixing a  $10^{-3}$  line and sharpening  $z_s$  to 3% and 1% moves the floor to 0.027 and 0.010. For scale, the surface potential  $\Phi_s/c^2 \sim 0.2\text{--}0.4$  sits 7.4 orders above the Sun-driven potential of terrestrial equivalence-principle laboratories ( $\sim 10^{-8}$ ) and 8.6 orders above Earth’s own surface potential ( $6.96 \times 10^{-10}$ ), the regime of MICROSCOPE; the spec-stage figure of  $10^{13}$  is not reproducible from any standard potential pair and is not used.

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Table 3: Shortlist source parameters with per-value provenance. Smearing  $\Delta E/E$  is at  $R_{\text{NS}} = 12$  km from Appendix C. Fields marked “from cyclotron” are inferred through Eq. (13). Values marked “(adopted)” are seeded-computation inputs, not audit-verified measurements.

| Source            | Spin                      | Field / cyclotron                                           | Flux / exposure context                                 | $\Delta E/E$            | Provenance notes                                                               |
|-------------------|---------------------------|-------------------------------------------------------------|---------------------------------------------------------|-------------------------|--------------------------------------------------------------------------------|
| EXO 0748–676      | 552 Hz [32]               | Low-field burster                                           | Burster; MINBAR sample [31]                             | $1.4 \times 10^{-1}$    | The disproved $z = 0.35$ source; fast spin is the disqualifier.                |
| Her X-1           | 1.24 s [10]               | $\sim 3 \times 10^{12}$ G, from cyclotron 37.4 keV [69, 70] | Line energy +6.5% per flux doubling [70]                | $2.0 \times 10^{-4}$    | Clears the wall; field- and flux-coupled line excludes it.                     |
| 4U 1626–67        | 7.66 s [10, 74]           | $\sim 4 \times 10^{12}$ G, cyclotron $\sim 36$ keV [74]     | $L = (3.3\text{--}4.9) \times 10^{36}$ erg/s [74]       | $3.3 \times 10^{-5}$    | Ongoing spin-down episode; epoch-specific spin.                                |
| GX 1+4            | $\sim 2$ min (epoch) [34] | Accreting magnetic pulsar                                   | $\sim 100$ mCrab historical; multi-year off states [34] | $\sim 2 \times 10^{-6}$ | Period drifts $\sim 50\%$ over decades; observability gaps.                    |
| Vela X-1          | $\sim 283$ s (adopted)    | Wind-fed high-field HMXB [10]                               | Persistent wind accretor                                | $9.0 \times 10^{-7}$    | Parameters adopted in the seeded computation.                                  |
| 1E 1207.4–5209    | Slow adopted (CCO;)       | Low inferred dipole field (anti-magnetar class)             | Reported surface absorption features                    | $2.0 \times 10^{-5}$    | Archetype low-field slow rotator; parameters adopted, not re-verified here.    |
| Magnetars (class) | 2–12 s [55]               | $B \gg 10^{14}$ G [42, 55]                                  | Variable; quiescent to outburst                         | $< 10^{-5}$             | Deepest wall evasion; catastrophically field-contaminated for a redshift line. |

from 4U 1820-303. *Astronomy & Astrophysics*, vol. 694, article A266. <https://doi.org/10.1051/0004-6361/202452878>. DOI: 10.1051/0004-6361/202452878.

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