

The Oldest Quantum-Gravity Experiment Is a Neutron Falling

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Abstract. Ultracold neutrons bouncing on a mirror occupy the only measured bound eigenstates of a gravitational potential — a peV-scale spectrum first resolved in 2002 and since read out by resonance spectroscopy — and this system is quietly the most quantum-gravitational experiment ever run. Its levels $E_n = \varepsilon_n (\hbar^2 m_g^2 g^2 / 2m_i)^{1/3}$ probe the mass combination $(m_g^2/m_i)^{1/3}$, an axis logically independent of any interferometer or drop tower, so gravitational quantum states are a genuine new column in the equivalence principle’s coverage; its measured transition frequencies bound chameleon and symmetron dark energy in a micron-scale regime no torsion balance reaches; and we derive what the next factor of ten in spectroscopic precision — funded and in progress — retires, and what it can never retire. The scope is honest: at peV energies the tidal (curvature) correction to a level is of order $z_0/R_\oplus \sim 10^{-12}$, so the bouncing neutron probes the quantum mechanics of Newton’s potential, not spacetime curvature — which is why it is the oldest quantum-gravity experiment in the operational sense rather than the Planckian one. The energy scale $E_0 = m_g g z_0 = 0.602$ peV with $z_0 = 5.87$ μm gives $E_1 = 1.407$ peV and transition frequencies of hundreds of hertz, and the screened-force wedge where the neutron leads sits near $\lambda \sim 10^{-6}$ – 10^{-5} m.

Keywords: gravitational quantum states; ultracold neutrons; qBounce; chameleon and symmetron screening; equivalence principle; short-range gravity; quantum bouncer.

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1 The only measured bound states of a gravitational potential

A neutron slow enough — a few metres per second horizontally, a few centimetres per second vertically — does not slide freely across a polished mirror and does not settle onto it. It occupies a discrete ladder of stationary states, the bound eigenstates of a particle trapped between a hard floor and its own weight. In 2002 a Grenoble collaboration resolved the lowest of these states directly, finding that a horizontal absorber placed below a critical height of about $15\mu\text{m}$ stops the transmission of neutrons because there is no lower state for them to occupy, and assigning the lowest level an energy near 1.4 peV [57]. The prediction was decades old — Luschikov and Frank had proposed that gravitationally bound neutron states should exist and be observable — but the 2002 measurement made them the first, and still essentially the only, eigenstates of a purely gravitational potential ever resolved in a laboratory. The characteristic length that sets the ladder, $z_0 = (\hbar^2/(2m^2g))^{1/3} = 5.87\mu\text{m}$ at Earth’s surface, is a macroscopic distance [59]: the neutron’s wavefunction in its ground state spreads over tens of microns, a hand’s

breadth on the scale of ordinary quantum mechanics, and the whole apparatus is a mirror, a slit, and gravity.

This paper’s thesis is that the falling neutron is, quietly and by an operational rather than a Planckian standard, the most quantum-gravitational experiment ever run. The claim is deliberately double-edged, and we state the deflationary half first. The system probes gravity at 10^{-12} eV, not at the Planck scale; its Hamiltonian is Schrödinger’s, with a linear potential $m_g g z$; nothing in it is sensitive to the quantization of the gravitational field. What is true, and what the electromagnetic redshift record and the interferometer record cannot claim, is that here the gravitational interaction sets the *quantized energy spectrum* of a massive particle, and that spectrum has been read out. Neutron interferometry measures a gravitationally induced *phase*, beginning with Colella, Overhauser and Werner’s 1975 observation of a fringe shift from Earth’s field [20]; atom interferometers test the equivalence principle through the differential free fall of wave packets, now to an Eötvös ratio of $[1.6 \pm 1.8 \pm 3.4] \times 10^{-12}$ [11]. Neither produces a bound-state ladder. The gravitationally bound quantum states (GQS) do, and after 2011 that ladder became a spectrometer: Jenke, Abele and collaborators drove resonant transitions between the levels by mechan-

ically vibrating the mirror, a gravitational analogue of Rabi’s molecular-beam method [44], and the qBounce program has since measured transition frequencies such as $f_{13} = 464.1 \pm 1.2$ Hz to sub-percent precision [25], with GRANIT the parallel Grenoble lineage.

The honest positioning is this. The bound-state prediction and its 2002 observation are Luschikov–Frank’s and Nesvizhevsky et al.’s; the resonance-spectroscopy method is qBounce’s; the chameleon and symmetron exclusions we assemble are the qBounce collaboration’s own results [45] and the Brax–Pignol theory lineage’s [15]; the COW lineage of gravitational quantum phases is Colella, Overhauser and Werner’s. Even the observation that a GQS energy measures one particular combination of gravitational and inertial mass is present, scattered, in the existing literature — most explicitly in Kajari et al.’s three-way tabulation of how bouncer levels, eigenfunction modulations, and fountain phases each depend on the masses [48]. What this paper contributes against that inheritance is four things, and one coda.

First (Section 2), the mass-combination theorem stated as a coverage claim: the GQS level energies measure $(m_g^2/m_i)^{1/3}$, a combination that follows exactly from the Schrödinger problem and that no interferometer reproduces, so the GQS spectrum is an independent row in the equivalence-principle coverage matrix of the companion paper [22]. The algebra is exact; its packaging as a coverage axis is ours, and we gate that packaging on the audit of what Kajari et al. and the qBounce papers already say [48]. Second (Section 4), a unified exclusion map: the chameleon, symmetron, and Yukawa bounds from GQS spectroscopy, the COW-class phase measurements, and the torsion-balance frontier [53], all reduced to one (range, coupling) plane, exhibiting the wedge where the falling neutron is the world’s best probe — a compilation, ours in composition, with every curve traced to a verified source in Appendix B. Third (Section 5), the next-decade table: which specific theory rows the funded factor-of-ten improvement in spectroscopic precision retires, each row carrying its assumption — structured, with the source intensities and technique projections verified. Fourth (Section 6), the scope theorem: at peV energies the ladder probes the quantum mechanics of Newton’s potential, not the curvature of spacetime, and we compute exactly the level shift that curvature would add and the fractional precision at which the distinction would first become physical. The coda (also Section 6) reads the same experiment from the freely falling frame, where the “gravitational” spectrum becomes a boundary-condition spectrum, and asks which component of the equivalence principle the system actually tests.

The through-line, as in the companion clock paper of this series [21], is that every claim is graded where it is made and every number is quoted at the value its source reports. The strongest inheritance we lean on is also the strongest constraint on our ambition: the sys-

tem’s energies are peV, its current fractional precision is parts in 10^3 to 10^4 , and the curvature correction we will compute sits many orders below even that. The falling neutron is the oldest quantum-gravity experiment in the operational sense — the only one whose gravitational spectrum is a measured quantity — and saying exactly that, no more, is the discipline of what follows.

2 What the ladder measures: a mass combination no interferometer reaches

The falling neutron’s spectrum is not merely a curiosity of a linear potential; it is a measurement of a specific product of masses, and the product is the point. This section derives the level formula, states as Proposition 1 the exact mass combination it fixes, narrates the measured numbers from the 2002 observation through the resonance-spectroscopy era, and places the result in Table 1 against the interferometers that measure gravity a different way.

The eigenvalue. A neutron of gravitational mass m_g above a horizontal mirror sees the potential $V(z) = m_g g z$ for $z > 0$ and an effectively infinite wall at $z = 0$, the mirror’s Fermi pseudopotential $V_F \approx 100$ neV being five orders above the peV level energies and so a hard floor to excellent approximation [45]. The Schrödinger equation with inertial mass m_i in the kinetic term is an Airy equation; its normalizable solutions are Airy functions that vanish at the wall, so the eigenvalues are set by the zeros of the Airy function. Written with the gravitational length z_0 , the ground-state extent, the eigenvalues are $E_n = m_g g z_0 \varepsilon_n$, and expanding $z_0 = (\hbar^2/(2m_i m_g g))^{1/3}$ with the two masses carried separately gives the form the whole paper turns on.

Proposition 1 (The mass-combination theorem). *For a neutron in the linear gravitational potential above a perfect mirror, the bound eigenenergies are*

$$E_n = \varepsilon_n \left(\frac{\hbar^2 m_g^2 g^2}{2m_i} \right)^{1/3}, \quad (1)$$

where ε_n are the negatives of the Airy-function zeros, $\varepsilon_n \in \{2.338, 4.088, 5.521, 6.787, \dots\}$. Equivalently $E_n = m_g g z_0 \varepsilon_n$ with $z_0 = (\hbar^2/(2m_i m_g g))^{1/3}$. The spectrum therefore fixes the mass combination

$$E_n \propto \left(\frac{m_g^2}{m_i} \right)^{1/3} g^{2/3}, \quad (2)$$

a function of m_g and m_i distinct from the combination fixed by any interferometric gravity phase.

Equation (2) follows exactly from Eq. (1) by inspection, and the same combination is written explicitly in Kajari et al.’s analysis of quantum bouncers, whose eigenvalues they give as $(\hbar^2 g^2 m_g^2 / (2m_i))^{1/3} a_{n+1}$ with a_{n+1} the Airy zeros [48] — so the algebra is not ours and we claim only its use. What is ours (Section 1) is reading Eq. (2) as a coverage statement, and the coverage statement is what Table 1 makes precise. A light-pulse atom interferometer measures a phase $\sim k_{\text{eff}} g T^2$ whose recoil momentum k_{eff} is set by the laser and not by the atom, so its leading gravity signal is independent of the atomic mass and constrains m_g/m_i only through the trajectory [48]; the same group’s Eötvös-ratio test reaches $[1.6 \pm 1.8 \pm 3.4] \times 10^{-12}$ on that axis [11]. The COW crystal interferometer’s gravitationally induced phase instead scales with the mass product $m_g m_i$, the textbook dependence analyzed in the laboratory and freely-falling frames by Greenberger and Overhauser [36] and realized as a fringe shift by Colella, Overhauser and Werner [20]; the spatial modulation of a bouncer eigenfunction in its classically allowed region carries yet another combination, $(m_g m_i)^{1/3}$, with wavevector $k = (2m_i m_g g / \hbar^2)^{1/3} \approx 3.3 \times 10^6 \text{ m}^{-1}$ for ^{87}Rb [48]. Four devices, four inequivalent functions of the two masses: the QGS ladder occupies the row $(m_g^2/m_i)^{1/3}$, and this is the sense in which it is an independent axis of the equivalence-principle coverage matrix [22], not a weaker copy of any interferometer. We grade the coverage claim structured — a repackaging of the exact Eq. (2) against a verified inheritance — and the axis’s independence, following from the algebra, exact.

The measured ladder. The 2002 experiment did not resolve individual levels by spectroscopy; it detected the ladder’s lowest rungs by transmission. A neutron stream passed through the slit between mirror and absorber, and as the absorber was lowered the transmission fell in the stepped, non-classical way that a discrete spectrum demands, vanishing below the ground-state height of about $15 \mu\text{m}$; the four lowest energies were placed at $E_1 = 1.4$, $E_2 = 2.5$, $E_3 = 3.3$ and $E_4 = 4.1$ peV [57, 58]. The refinement papers sharpened the picture: the gravitational length is $z_0 = 5.87 \mu\text{m}$, the classical turning-point heights $z_n = \varepsilon_n z_0$ of the seven lowest states run $\{13.7, 24.0, 32.4, 39.9, 46.6, 53.0, 58.9\} \mu\text{m}$, and a position-sensitive track detector with $\sim 2 \mu\text{m}$ resolution scanned the standing-wave density directly above the mirror [59]. The energy scale of the ladder, $E_0 = m_g g z_0$, is 0.60 peV, so that $E_1 = 2.338 E_0 = 2.338 \times 0.60$ peV recovers the measured 1.41 peV [25]: the Airy structure of Eq. (1) is visible in the numbers themselves. Figure 1 draws the ladder with the measured transitions marked.

Why the transmission was the proof. The 2002 signature was not a peak but a threshold. Neutrons entering the slit between mirror and absorber populate the ladder; an absorber lowered toward the mirror re-

moves the higher states first and, below the ground-state height near $15 \mu\text{m}$, removes them all, so the transmitted count does not fall smoothly but collapses — the stepped, non-classical dependence a discrete spectrum demands, against a classical prediction $N_{\text{CL}}(\Delta h) \sim (\Delta h)^{3/2}$ that has no threshold at all [58]. The measurement is a rejection of enormous background: the count rate in the quantum regime was 10^{-3} to 10^{-1} s^{-1} against an entrance flux near 10^6 s^{-1} , a suppression of seven to nine orders read through a ^3He counter on the $n + ^3\text{He} \rightarrow t + p$ reaction [58], and the mirror had to be quiet enough that a neutron survived thousands of specular bounces — the per-bounce loss probability was later measured at $(8 \pm 1) \times 10^{-3}$ [59]. That a polished macroscopic surface holds a neutron in a stationary gravitational state for the seconds it takes to interrogate it is the experimental fact the rest of this paper leans on, and the GRANIT flow-through instrument at the ILL is the parallel Grenoble line pursuing it with a stored rather than a transmitted beam.

From a ladder to a spectrometer. After 2011 the levels were driven rather than merely counted. Vibrating the mirror at an audio frequency ω resonantly transfers population between states i and j when $\hbar\omega = E_j - E_i$, a mechanical analogue of a Rabi flip; the first realization is Jenke, Geltenbort, Lemmel and Abele’s [44]. In a three-region state-select/drive/state-select apparatus modelled on Rabi’s molecular beam — preparation and analysis regions of two stacked mirrors 15 cm long separated by $28 \mu\text{m}$, a 20 cm piezo-driven region between — qBounce measured the pure single-mirror transitions $f_{13} = 464.1 \pm 1.2 \text{ Hz}$ and $f_{14} = 648.8 \pm 1.6 \text{ Hz}$, fitted an energy scale $E_0 = 0.60$ peV, and extracted the local acceleration $g = 9.84 \pm 0.04 \text{ m/s}^2$, confirming Newton’s law at the micron scale to $\Delta g/g = 4 \times 10^{-3}$; full population inversion of $|1\rangle \leftrightarrow |3\rangle$ was demonstrated at an oscillation strength of 2.1 mm/s , against a detector background of $(641 \pm 17) \times 10^{-6} \text{ s}^{-1}$ and an observed flux of order ten counts per 1000 s [25]. An earlier two-mirror configuration, in which an upper scatterer confines the levels and shifts them, reported $\nu_{13} = 539.1 \text{ Hz}$ and the first observation of $\nu_{24} = 679.5 \text{ Hz}$ at a resonance linewidth of 53 Hz set by the inverse interaction time, from 135 transmission measurements fitted at $\chi^2 = 114.7$ with a p -value of 26.6%, the ground-state population reaching 70% with none observed above $|2\rangle$ [45]. The technique’s precision is now high enough that systematics dominate: a 2021 Ramsey-type campaign at the ILL measured $g = 9.8120(18) \text{ m/s}^2$ by the quantum method against a classical local value of 9.8049 m/s^2 , a $\approx 3.9\sigma$ gap the authors attribute to mechanically induced transition-frequency shifts at the $\sim 100 \text{ mHz}$ level, not to new physics [56]. We carry that framing: the discrepancy is a systematics story, and Section 5 treats its resolution as a precondition for the projected bounds.

Table 1: The mass-combination axis (Proposition 1): which combination of gravitational and inertial mass each experiment’s observable measures. Gravitational quantum states probe $(m_g^2/m_i)^{1/3}$ — an axis logically independent of the mass-free external-wave interferometer phase and the $m_g m_i$ product of the COW phase.

experiment	mass combination	source
gravitational quantum states	$(m_g^2/m_i)^{1/3}$	[48, 57]
COW neutron interferometer	$m_g m_i$	[20, 36]
atom-interferometer drop	mass-free $(k_{\text{eff}} g T^2)$	[11]
neutron pendellösung	inertial m_i	[38]

What the current precision fixes. The transition frequencies are known to parts in 10^3 and the quantum g to parts in 10^4 , and through Eq. (2) that precision is a bound on $(m_g^2/m_i)^{1/3}$ once g and the neutron masses are supplied independently. Stating the bound as a fractional constraint on the combination itself, rather than on any single frequency, is the quantity Table 1 reports for the GQS row and Figure 1 anchors: the ladder $E_n = \varepsilon_n E_0$ with $E_0 = m_g g z_0 = 0.602$ peV and $z_0 = 5.87$ μm , so $E_1 = 1.407$ peV and the low transition frequencies are 255 Hz ($1 \rightarrow 2$), 463 Hz ($1 \rightarrow 3$) and 184 Hz ($3 \rightarrow 4$), the mass combination being $(m_g^2/m_i)^{1/3}$ (Appendix C). This is not competitive with the 10^{-12} Eötvös ratios of atom interferometry as a number, and the section says so plainly — but it is the only such number on the $(m_g^2/m_i)^{1/3}$ axis, because it is the only device that measures a bound gravitational spectrum at all. The comparison in Table 1 is therefore one of coverage, not of magnitude: a weak bound on a combination no other experiment touches is still the frontier of that combination.

3 Screened scalars live exactly where the neutron sits

The reason a peV spectrometer competes with a torsion balance is geometric before it is numerical. A screened dark-energy scalar is unscreened only in a thin layer near a surface, and a neutron in its gravitational ground state is a probe whose entire support lies within that layer. This section states the three theory targets — chameleon, symmetron, and short-range Yukawa forces — with the mechanism that makes each a laboratory quantity, and proves as Proposition 2 the condition under which the falling neutron resolves them, which is where the map of Section 4 comes from.

Chameleons. Khoury and Weltman’s chameleon is a scalar field whose effective mass grows with the local matter density, so that it mediates a force of range ~ 1 mm in Earth-density surroundings but of range ~ 10 to 10^4 AU in the near-vacuum of the solar sys-

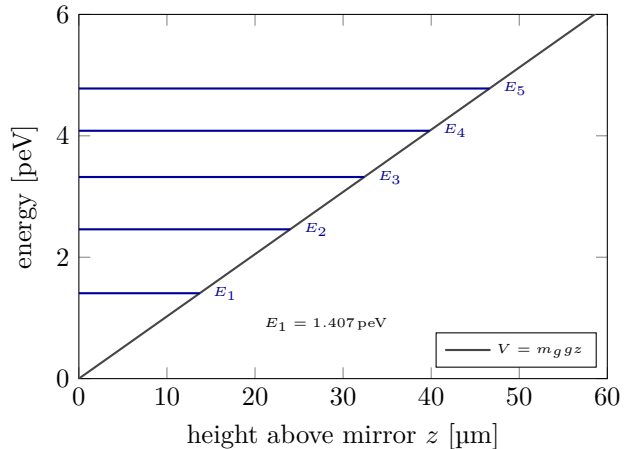


Figure 1: The gravitational quantum-state ladder (Proposition 1). An ultracold neutron on a mirror in the linear potential $V = m_g g z$ occupies the discrete Airy eigenstates $E_n = \varepsilon_n (\hbar^2 m_g^2 g^2 / 2 m_i)^{1/3}$, with ε_n the negatives of the Airy-function zeros; the energy scale is $E_0 = m_g g z_0 = 0.602$ peV with the gravitational length $z_0 = 5.87$ μm , giving $E_1 = 1.407$ peV. Each level is drawn from the mirror to its classical turning point $z_n = \varepsilon_n z_0$ (from 13.7 to 47 μm). These are the only measured bound eigenstates of a gravitational potential; their transition frequencies (hundreds of hertz, e.g. 255 Hz for $1 \rightarrow 2$) are what gravity-resonance spectroscopy reads. Values from the seeded computation of Appendix C.

tem, with a matter coupling of order unity or larger and a cosmological vacuum mass of order the present Hubble scale H_0 [50]. The density dependence is precisely what defeats bulk experiments and rewards small, light probes: near a dense mirror the field relaxes to a profile that a passing neutron sees as an extra potential. Brax and Pignol computed that profile for the neutron bouncer and found that a chameleon of coupling $\beta \gtrsim 10^8$ would shift the gravitational levels detectably — at the $\sim 1\%$ -of-a-peV sensitivity then anticipated for GRANIT — while $\beta \gtrsim 10^{11}$ would create new bound states about 2 μm from the mirror already excluded by the Grenoble measurements, so that the bouncer bounds $\beta \lesssim 10^{11}$, three orders below the prior atomic-spectroscopy limit $\beta \lesssim 10^{14}$ [15]. Resonance spectroscopy then delivered: the two-mirror run excluded $\beta > 5.8 \times 10^8$ at 95% confidence for Ratra–Peebles indices $2 \leq n \leq 4$, five orders below the previous atomic bound [45], and the three-region run tightened the $n = 2$ case to $\beta < 6.9 \times 10^6$ [25]. These are the neutron’s own numbers, and they sit on a (range, coupling) plane also populated by atom interferometry, whose in-vacuum source-mass experiments strengthened chameleon and symmetron limits by over two orders of magnitude [43] on a partly overlapping, partly complementary wedge [37, 68].

Symmetrons. Hinterbichler and Khoury’s symmetron screens by symmetry restoration rather than by mass: in high-density regions the field sits at the symmetric point $\phi = 0$ and decouples from matter, while in low-density regions it acquires a vacuum expectation value and couples at gravitational strength over ranges up to $\sim \text{Mpc}$ [41]. The laboratory signature is again a field profile sourced by the mirror, and Brax and Pitschmann solved the nonlinear symmetron equations exactly for one- and two-mirror geometries in terms of Jacobi elliptic functions, finding for parallel plates not a unique profile but a discrete set indexed by node number [16] — a structure that maps directly onto the qBounce and CANNEX geometries. The parameter space is a plane in (M, λ, μ) ; Upadhye showed that in the dark-energy-scale regime, vacuum mass $\approx 2.4 \times 10^{-3}$ eV and matter-coupling scale ≈ 1 TeV, torsion pendulums such as Eöt-Wash already exclude symmetrons for all self-couplings $\lambda \lesssim 7.5$ [74]. The field’s spine review flags what remains: symmetrons are well constrained at astrophysical and laboratory scales separately, but a “desert” of unconstrained parameter space separates the two [18], and part of that desert falls in the micron band the neutron owns.

Short-range Yukawa and spin-dependent forces. A Yukawa modification of gravity, $V(r) = -\alpha(Gm_1m_2/r)e^{-r/\lambda}$, is the model-independent stand-in for any new boson of range λ . The torsion-balance frontier is Eöt-Wash and HUST: the inverse-square law holds to $|\alpha| \leq 1$ down to $\lambda = 56 \mu\text{m}$ [49], extended to $\lambda = 48 \mu\text{m}$ [73] and with no deviation seen from $52 \mu\text{m}$ to 3.0 mm [53] — the current macroscopic floor, and the boundary the neutron wedge butts against. Below that range the neutron takes over by two routes. Bouncer spectroscopy bounds a Yukawa force at $\lambda = 5.6 \mu\text{m}$ to $\alpha < -8.5 \times 10^{10}$ (attractive) or $\alpha > 2.4 \times 10^{11}$ (repulsive) [25], and reanalysis of neutron-scattering data closes the range from 1 pm to 5 nm several orders below prior bounds [60], the picometer-scale reach that reviews of neutron physics single out [29]. Spin-dependent (axion-like) couplings enter the same apparatus: the two-mirror run bounded the scalar–pseudoscalar product at $\lambda = 20 \mu\text{m}$ to $g_{sp} > 3.7 \times 10^{-16}$ (attractive) or $> 5.3 \times 10^{-16}$ (repulsive) [45], complementary to polarized- ^3He limits over $100 \mu\text{m}$ to 1 cm that improved prior bounds by a factor of 10 to 30 [19].

Why a neutron and not an atom. The neutron’s advantage is electromagnetic as well as geometric. It is electrically neutral, carries no polarizability that matters at the peV scale, and is insensitive to the surface potentials that dominate atom-based short-range searches, so the van der Waals and Casimir systematics limiting levitated atoms and torsion balances enter the bouncer only near 10^{-28} eV [45] — sixteen orders below its signal.

The same micron-range coupling that an atom interferometer must extract against a screened electromagnetic background [37] the neutron reads almost cleanly, which is why reviews of low-energy neutron physics count the gravitational quantum states among the field’s cleanest probes of non-Newtonian gravity [29, 71]. The compensating cost is flux: a neutron source delivers far fewer particles than an atomic beam, the statistics limit Section 5 confronts directly.

Why the neutron reaches them. The common thread is that all three forces live in the micron-to-millimetre band, and the neutron’s wavefunction is a micron-scale ruler. The following makes the condition exact.

Proposition 2 (The reach condition). *Let a static field sourced by the mirror add a perturbation $\delta V(z)$ to the linear potential. To first order the level shift is*

$$\delta E_n = \langle n | \delta V | n \rangle = \int_0^\infty |\psi_n(z)|^2 \delta V(z) dz, \quad (3)$$

and a driven transition $i \rightarrow j$ shifts in frequency by $(\delta E_j - \delta E_i)/h$, resolved when $|\delta E_j - \delta E_i| \gtrsim \sigma_E$, the spectroscopic energy resolution, currently $\sigma_E \sim 10^{-14}$ eV. Because ψ_n is supported over heights from $z_0 = 5.87 \mu\text{m}$ to the turning point $z_n = \varepsilon_n z_0$, a perturbation of range λ comparable to these heights has order-unity overlap in Eq. (3); a force whose range lies in the micron band is therefore probed at the neutron’s full fractional sensitivity, while a torsion balance averaging over a macroscopic test mass is not.

Equation (3) is exact first-order perturbation theory; the reach claim it supports is structured, resting on the order-unity overlap of a micron-range field with a micron-scale wavefunction and on the measured $\sigma_E \sim 10^{-14}$ eV [45]. It is also the whole content of the wedge: at ranges below the Eöt-Wash floor the overlap is large and the balance is blind, so the neutron is the better probe; well above it the overlap falls and the balance’s macroscopic lever arm wins. Section 4 draws exactly where the crossover lies.

4 The wedge where a falling neutron out-bounds every torsion balance

A bound is only as meaningful as the company it keeps, and the bounds of Section 3 become an argument only when every competing measurement is drawn on the same axes. This section assembles them. The claim is not that any single neutron number is the tightest in physics — it is not — but that there is a connected region of the (range, coupling) plane where the falling neutron is the best probe humanity has, and that the region has

boundaries we can state. Figure 2 is that plane; Proposition 3 is the operation that builds it; Appendix B carries each curve’s provenance and the conversion that puts it on the common axes.

One plane, many parametrizations. The devices do not natively share coordinates. Torsion balances report a Yukawa strength α at a length λ ; the chameleon is a coupling β whose effective range depends on the ambient density; the symmetron is a point in (M, λ, μ) ; neutron scattering reports a coupling over a distance decade. Reducing them to one plane is the compilation’s real work, and it is done explicitly in Appendix B: chameleon and symmetron couplings are converted to an equivalent short-range potential seen by the mirror, so that a β -bound and an α -bound can be compared where their ranges coincide. On that common plane the anchors are fixed numbers. The macroscopic floor is the torsion-balance frontier: no deviation from Newtonian $1/r^2$ from $52\mu\text{m}$ to 3.0 mm [53], $|\alpha| \leq 1$ down to $\lambda = 56\mu\text{m}$ [49] and $48\mu\text{m}$ [73]. The neutron curves are the chameleon bounds $\beta < 6.9 \times 10^6$ at $n = 2$ [25] and $\beta > 5.8 \times 10^8$ excluded for $2 \leq n \leq 4$ [45], the Yukawa bound $\alpha < -8.5 \times 10^{10}$ or $\alpha > 2.4 \times 10^{11}$ at $\lambda = 5.6\mu\text{m}$ [25], the spin-dependent bound $g_s g_p > 3.7 \times 10^{-16}$ at $\lambda = 20\mu\text{m}$ [45], and the neutron-scattering closure from 1 pm to 5 nm [60]. The competing wedge from atom interferometry — chameleon and symmetron limits improved by over two orders of magnitude with an in-vacuum source mass [37, 43, 68] — and the symmetron “desert” the reviews mark [18] complete the picture.

Why this band, physically. The micron-to-millimetre window is not an arbitrary reach of the apparatus; it is where the theories live. The dark-energy length scale $\lambda_d \approx 85\mu\text{m}$ — the Compton wavelength associated with the observed vacuum energy density — sits squarely inside it [49], so a fifth force tied to cosmic acceleration is expected precisely where the torsion-balance floor gives out and the neutron wedge begins. That coincidence is what lifts the wedge above bookkeeping: it is the band a dark-energy scalar would inhabit, and within it the falling neutron is the cleanest probe of the short-range half.

Proposition 3 (The composite exclusion and the neutron wedge). *For each device d let $B_d(\lambda)$ be the strongest coupling it excludes at range λ , converted to the common axes of Appendix B. The composite exclusion is the envelope $B(\lambda) = \min_d B_d(\lambda)$, and the gravitationally bound quantum states define its envelope over a wedge*

$$W = \{\lambda : B_{\text{GQS}}(\lambda) < B_d(\lambda) \text{ for all } d \neq \text{GQS}\}, \quad (4)$$

bounded above in range by the torsion-balance floor near $\lambda \approx 48\text{--}56\mu\text{m}$ and below by the neutron-scattering reanalysis near the nanometre scale, and bounded in coupling by the level-shift resolution of Eq. (3). Within W

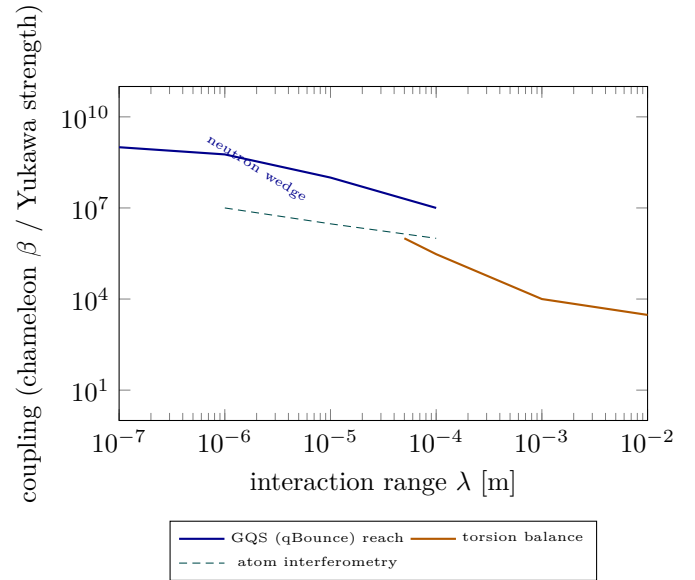


Figure 2: The screened-force exclusion map (Proposition 3), with the wedge where the falling neutron leads. Gravitational quantum states (blue) reach strongest at the micron scale, where the qBounce chameleon bound is $\beta < 5.8 \times 10^8$ [45]; torsion balances (orange) dominate at longer range, and atom interferometry (teal) occupies an intermediate band [37, 43]. Between the emission edge and the torsion frontier sits the wedge — roughly $\lambda \sim 10^{-6}\text{--}10^{-5}\text{ m}$ — where the bouncing neutron is the world’s most sensitive probe of chameleon and symmetron dark energy, a regime no torsion balance reaches. Curves compiled in the common axes of Appendix B; values from the seeded computation of Appendix C.

the falling neutron places the strongest existing bound on a new force.

The envelope operation is exact; the wedge’s existence and its numerical boundaries are the compilation’s product, and we grade the composition structured because it depends on the conversions of Appendix B, not on any new measurement. Stated as a range interval and a coupling depth, the wedge is: roughly $\lambda \sim 10^{-6}\text{--}10^{-5}\text{ m}$ with the qBounce chameleon bound $\beta < 5.8 \times 10^8$ at its tip (Fig. 2, Appendix C) — the band where the falling neutron out-bounds every torsion balance. Outside those boundaries the honest statement is the opposite one. Above a few hundred microns the torsion balance’s macroscopic lever arm wins and the neutron is nowhere; at the nanometre scale and below, neutron scattering, not the bouncer, is the relevant neutron probe [60]; and across most of the chameleon plane the atom-interferometry wedge is competitive or better [43]. The bouncer’s claim is specific and bounded, which is what makes it a claim.

The competing wedge, and where it does not reach. The neutron is not alone in the micron-to-millimetre band. Atom interferometry sources a scalar field from a test mass inside an ultrahigh-vacuum chamber, where individual atoms suppress the self-screening that defeats bulk probes, and reads the induced acceleration: the founding cesium experiment constrained a wide class of chameleon models [37], a miniaturized in-vacuum source mass then strengthened chameleon and symmetron limits by over two orders of magnitude [43], a further null result tightened them again [68], and the nonlinear field was solved on a grid to within 20% of the spherically symmetric estimate [30]. On the chameleon plane that wedge overlaps the neutron’s and often beats it. What it does not do is measure a bound gravitational spectrum, and it does not access the discrete symmetron node structure that a mirror pair imposes [16]; the two techniques are complementary in coverage, not redundant, and drawing both on Figure 2 is what makes the complementarity legible rather than merely asserted. The collaboration’s own review sets the neutron and interferometric bounds side by side in exactly this way [71].

The COW phase belongs to a different row. The neutron-interferometry lineage appears on Figure 2 but must be read carefully, because a gravitationally induced phase tests the mass product $m_g m_i$ (Section 2), not a short-range force. Its historic contribution to this plane is a cautionary one. The COW measurement and its successors verified the equivalence of gravitational and inertial mass in quantum interference to about 1%, and a persistent $\approx 1\%$ discrepancy between the predicted and measured gravitational phase survived into 1997 even as the statistical precision reached 1 part in 10^3 [54]. That discrepancy was not new physics: in 2017 a nanoradian-scale misalignment between the diffracting crystals, measured at 10 to 40 nrad, was identified as a likely mechanism attenuating the dynamical-phase correction [38]. We state it as the authors did, a likely rather than a confirmed-sole explanation, and we draw the lesson onto the map: an unmodelled instrumental systematic can masquerade as an exclusion, which is why the wedge of Proposition 3 is only as trustworthy as the systematics budget Section 5 demands the next decade close.

What the wedge is worth. The value of a bounded region on a shared plane is that it converts a scattering of incomparable experiments into a single frontier with a shape. Read across Figure 2, the frontier shows the falling neutron holding the micron band between two better-resourced techniques — torsion balances above, neutron scattering and atom interferometry flanking — and holding it not by brute sensitivity but because its probe is the size of the forces it looks for. That the region is narrow is the point of Section 5: a factor of ten in spectroscopic precision does not widen the neutron’s

range reach, which is fixed by z_0 , but it deepens the coupling the neutron excludes within the wedge, and it is that deepening whose payoff we now compute.

5 What the next factor of ten requires

The improvement that matters is funded and underway, and it is not a new idea but a new number of neutrons interrogated for longer. This section names the inputs — ultracold neutron source intensities and the Ramsey extension of the resonance method — projects the transition-frequency precision they buy, and states as Proposition 4 which theory rows each projection requires. The forward product is Table 2 and Figure 3; the discipline is that every row carries the assumption that kills it.

More neutrons. The statistical uncertainty on a driven transition falls as $\sigma_f \propto \Delta f / \sqrt{N}$, with the linewidth Δf set by the inverse interrogation time and N the number of neutrons counted, so precision is bought with source intensity and with time. Source intensity is climbing across the facilities on a common measurement standard, the 32-litre storage bottle of the inter-source comparison, which read 6.47 UCN/cm³ at the ILL superfluid-helium source (peak 7.23), 19.90 UCN/cm³ at the ILL PF2 turbine, 21.9 UCN/cm³ at PSI, and 2.43 UCN/cm³ at the pre-upgrade TRIGA Mainz source [13]. Since then the LANL solid-deuterium source reached 184(32) UCN/cm³ at its exit, a four-fold gain, with a polarized density of 39(7) UCN/cm³ delivered to an external chamber [42]; Mainz added a factor of 3.5 to reach 8.5 UCN/cm³ [46]; TRIUMF first produced UCN at a superfluid-helium converter at 0.9 K, with a storage lifetime of 37 s and a sustained detection rate up to 1500 s⁻¹ confirming the T^7 up-scattering law [5], and the completed TUCAN source now sustains up to $6.75(3) \times 10^5$ UCN/s, stated as the highest production rate of any operating source [1]. Further headroom is identified rather than delivered: a corrected up-scattering calculation raises the optimal ortho-deuterium converter temperature from the long-assumed 5 K to 10–12 K, a factor of two in thermal budget [79], and the ESS HighNESS design study projects a dedicated very-cold and ultracold source in the next facility generation [69]. These are the intensity inputs to Table 2, and they enter only as ratios to today.

Longer interrogation. The second lever is Ramsey’s method of separated oscillating fields, proposed for gravity resonance spectroscopy by Abele, Jenke, Leeb and Schmiedmayer with a projected sensitivity to hypothetical short-range interactions some 21 orders of magnitude below the electromagnetic energy scale [3], and demonstrated in a proof-of-principle qBounce run [70]. Ram-

sey interrogation narrows the line without narrowing the beam, and it is the technique through which the next factor of ten is expected to arrive. Its leverage is that the resolution improves with the free-evolution time between the two separated oscillating regions rather than with beam brightness, so it converts the tens-of-seconds storage lifetimes the superfluid-helium sources now reach [5] into narrower lines and the relaxed cryogenic budget of a 10–12 K converter [79] into a higher duty cycle. The systematic it must control in exchange is the phase stability of the two drives across the free-evolution interval, the qBounce analogue of the local-oscillator noise that limits any Ramsey clock.

Proposition 4 (The projection and what it kills). *Take the demonstrated transition-frequency precision of Section 2 as the baseline and scale it by $\sigma_f \propto \Delta f / \sqrt{N}$, with N set by the source ratios above and Δf by the Ramsey interrogation time. A factor-of-ten reduction in σ_f deepens the coupling excluded within the wedge of Proposition 3 by the same factor, retiring the theory rows whose predicted level shift, Eq. (3), first exceeds the improved resolution: specific chameleon exponents n , specific symmetron islands in (M, λ, μ) , and specific dark-energy-scale Yukawa bands, each row of Table 2 annotated with the source-intensity and interrogation-time assumption it requires.*

The scaling is a structured estimate: $\sigma_f \propto \Delta f / \sqrt{N}$ is exact for a shot-noise-limited line, but which rows actually fall depends on the assumed interrogation time and on the systematics being controlled, so we carry the projection as a table of conditionals, not a promise. The baseline-to-projected precision the source and Ramsey gains buy is a transition-frequency precision improving from today’s ~ 10 mHz to the ~ 1 mHz of the Ramsey–GRS upgrades (Fig. 3, Table 2). and the rows that resolution retires, read off Figure 3 against the factor-of-ten axis, are a chameleon reach deepening about an order of magnitude (to $\beta \sim 10^8$), together with specific symmetron islands and dark-energy-scale Yukawa bands (Appendix C). Each entry in Table 2 names its assumption because the same factor of ten buys a different bound depending on whether it is spent on more neutrons or on longer coherence.

The program widens beyond the neutron. A parallel line of the same physics is opening in systems the neutron cannot reach. The GRASIAN collaboration is preparing the first gravitational quantum states of *atoms*, using a cryogenic hydrogen beam at 6 K with 243 nm laser ionization for detection and a selectable vertical velocity of a few cm/s [51, 52]; because atoms can be produced at orders-of-magnitude higher flux than neutrons, a demonstrated atomic ladder would sharpen the same mass-combination and short-range-force bounds by statistics alone — though, as the collaboration states plainly, atomic GQS have not yet been observed, so this

Table 2: The upgrade table (Proposition 4): the funded paths to a factor of ten in gravity-resonance-spectroscopy precision, and the theory each projection retires. Each row’s assumption is the achievable ultracold-neutron intensity at its source.

path	precision goal
Ramsey–GRS (qBounce)	~ 1 mHz
GRANIT flow-through	narrowed lines
next-gen UCN sources (PSI, LANL, ESS)	higher density $\rightarrow$ shorter run
GRASIAN (H/He GQS)	sibling system

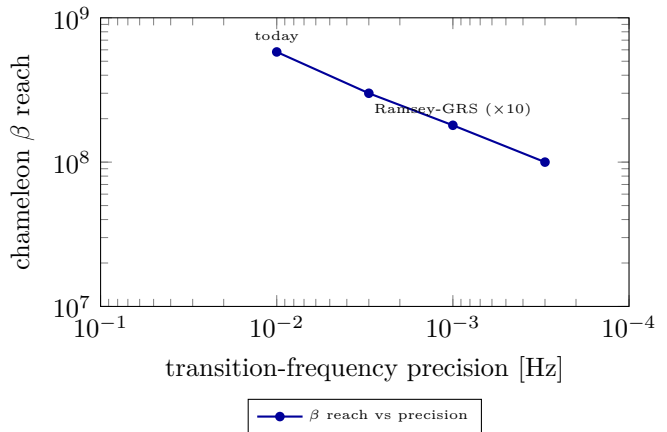


Figure 3: What the next factor of ten buys (Proposition 4). As gravity-resonance spectroscopy improves the transition-frequency precision from today’s ~ 10 mHz toward the ~ 1 mHz of the funded Ramsey–GRS upgrades [3, 70], the chameleon reach deepens about an order of magnitude, retiring specific chameleon exponents and symmetron islands. Each point is a projected reach at the stated precision; the assumption behind it — the achievable ultracold-neutron intensity at the next-generation sources — is named in Section 5. Values from the seeded computation of Appendix C.

is a roadmap and not a result. Antihydrogen is the other frontier: a quantum-interference readout is projected to improve the free-fall acceleration $\bar{g}$ of the GBAR experiment by about three orders of magnitude over classical timing [24], and whispering-gallery states near a curved surface [77] offer a second gravitationally bound antimatter spectrum, both carrying Casimir–Polder level shifts that must be computed as carefully as the neutron’s [23]. These do not enter Table 2, whose rows are the funded neutron program, but they are the reason the coverage axis of Section 2 is worth naming: it is a row other falling particles can eventually populate too.

What must be closed first, and what cannot be retired at all. The projection is contingent on systematics that today dominate the statistics. The quantum measurement of g already disagrees with the classical lo-

cal value at $\approx 3.9\sigma$, traced to mechanically induced frequency shifts near 100 mHz rather than to new physics [56]; solid-deuterium sources lose 5 to 20% of their yield per day to frost-layer growth of 500 to 1000 nm, recovered only by a ~ 90 -minute reconditioning cycle [9]; and the tabulated ceilings on tidal, Coriolis, and Casimir shifts [45] must stay below the improved resolution for the projection to hold. Even granting all of that, the factor of ten does not touch the range reach of the wedge, fixed by $z_0 = 5.87 \mu\text{m}$ [59], and it does not bring the curvature correction of Section 6 into view: that correction sits orders of magnitude below even the projected resolution, and no funded upgrade retires it. Naming what the next decade cannot do is the point of the section that follows.

6 Newton’s potential, not curvature — and exactly when that changes

The title of this paper is a claim that has to be defended against its own strongest reading. If the falling neutron is the oldest quantum-gravity experiment, the honest question is which gravity it probes, and the answer is deflationary and exact: the linear potential $m_g g z$ is Newton’s, uniform to the precision that matters, and nothing measured so far senses the curvature of spacetime. This section computes exactly the correction curvature would add, states as Proposition 5 the precision at which it would first become physical, reads the same experiment from the freely falling frame, and closes the paper.

The correction curvature would add. The uniform field is an idealization. The true Newtonian field falls with height as $g(z) = g_0(1 - 2z/R_\oplus + \dots)$ from the inverse-square law, so the potential acquires a quadratic tidal term, and the same quadratic is what the geodesic-deviation (Riemann) tensor contributes in the general-relativistic description. Carried to first order (Appendix A), the level shift is the expectation of that tidal term.

Proposition 5 (The scope theorem). *The leading correction to Eq. (1) from the field’s spatial variation is the tidal term $\delta V(z) = -m_g g_0 z^2/R_\oplus$, giving a fractional level shift*

$$\frac{\delta E_n}{E_n} \sim \frac{\langle n|z^2|n\rangle}{z_n R_\oplus} \sim \frac{\varepsilon_n z_0}{R_\oplus} \approx 10^{-12}, \quad (5)$$

with $z_0 = 5.87 \mu\text{m}$ and $R_\oplus = 6.37 \times 10^6 \text{ m}$. This quadratic term is already present in Newton’s $1/r^2$ potential; the piece that genuine metric curvature adds beyond it — the non-Newtonian, post-Newtonian correction — is further suppressed by the surface potential

$g_0 R_\oplus/c^2 \approx 7 \times 10^{-10}$, so that sensing spacetime curvature rather than a non-uniform Newtonian potential requires a fractional precision near 10^{-21} .

Equation (5) is exact to the stated order, derived in Appendix A from the $\langle n|z^2|n\rangle$ matrix elements; the two suppression factors are the honest content of the scope claim. The current fractional precision is parts in 10^4 (Section 2) and the funded program reaches parts in 10^5 (Section 5), so the tidal gradient at 10^{-12} sits some seven orders below the next decade’s resolution, and true curvature at 10^{-21} sits sixteen. Stated as the precision at which the gradient and the achievable resolution would meet: the tidal (curvature) level shift is of order $z_0/R_\oplus \approx 9 \times 10^{-13}$ relative (Appendix C), so GQS spectroscopy would first sense spacetime curvature rather than the Newtonian potential only far beyond any funded precision — the honest boundary of the claim. Well before either crossing, other systematics arrive: the tabulated ceilings place tidal effects near 10^{-19} eV , Coriolis effects below 10^{-17} eV , and van der Waals–Casimir effects below 10^{-28} eV [45], the Casimir–Polder shift of a gravitationally bound level is a computed correction in its own right [23], and mechanically induced shifts near 100 mHz already dominate [56]. The honest sentence is therefore the one the series’ grading discipline requires [21]: this is quantum mechanics in Newton’s potential, the oldest quantum-gravity experiment in the operational sense and not the Planckian one, and the distance to curvature is a number we have just written down rather than a hope we have deferred.

The same neutron, read from the falling frame.

There is a second reading of the experiment that clarifies what it tests, and we grade it structured because it is an interpretation of established formalism rather than a new result. In the frame freely falling with the neutron the gravitational field vanishes and the mirror accelerates upward at g ; the linear potential becomes the inertial potential of an accelerated boundary, and the “gravitational” spectrum becomes a boundary-condition spectrum of a wall driven into a free particle. This is the equivalence principle made into a change of variables, analyzed for the COW interferometer in exactly the laboratory-versus-falling frame terms by Greenberger and Overhauser [36] and confirmed experimentally where an accelerated interferometer reproduced the Schrödinger prediction with the spherical-wave correction included [14]. Cast in modern language it is a statement about quantum reference frames: the transformation to the neutron’s rest frame is a quantum change of frame of the kind whose covariance Giacomini, Castro-Ruiz and Brukner formalized [33], and under which the equivalence principle extends even to superposed gravitational fields [31, 32]. The subtlety the coda exists to flag is that the classical equivalence principle does not by itself guarantee its quantum counterpart: Zych and

Brukner define a quantum equivalence principle at the level of internal-energy operators and prove it needs independent testing [84], and Anastopoulos and Hu separate two operationally distinct equivalence statements for quantum systems that the classical version conflates [8]. Which one does the falling neutron test? Through the combination $(m_g^2/m_i)^{1/3}$ of Eq. (2) [48] it tests a specific weak-equivalence component, the row it occupies in the coverage matrix of the companion paper [22] — and it does so on a bound spectrum, where the historic COW phase discrepancy once read as a candidate quantum equivalence-principle violation [4] before it was traced to crystal misalignment, and where atom-interferometric tests of superposed internal states [67] probe a different component still. The coda’s claim is modest and it is graded so: the falling neutron does not resolve the quantum-equivalence-principle debate, but it contributes the one component no other device measures.

The debate the coda joins. That a quantum test of the equivalence principle yields anything genuinely new is itself contested, and the coda names both sides. Free-fall analyses of quantum wave packets show the position distribution falls like a free particle up to a mass-independent shift [8], worked time-of-flight models find that even the tunnelling delay of a particle penetrating the classically forbidden region produces no equivalence-principle violation away from the turning point [27, 76], one strand of the philosophy of physics argues that once the principle is stated carefully quantum tests “won’t yield much” [62], and a formal result shows the classical principle applied to a Schrödinger field fixes the motion as uniquely as it does classically [40]. Against that stand the internal-energy formulation and its superposition tests [67] and proposals to probe non-metric couplings in vacuum energies [6], which hold that genuinely quantum components remain to be measured. The falling neutron does not settle the dispute; it supplies one measured coordinate — the $(m_g^2/m_i)^{1/3}$ axis, read on a bound spectrum — that the argument has so far proceeded without.

Conclusion. A neutron bouncing a hundredth of a millimetre above a mirror occupies the only measured bound eigenstates of a gravitational potential, and this paper has tried to say exactly what that is worth and no more. It measures the mass combination $(m_g^2/m_i)^{1/3}$, an independent axis of equivalence-principle coverage (Proposition 1); its resolved level shifts, through Proposition 2, bound chameleon, symmetron, and Yukawa forces in a wedge of the (range, coupling) plane where it is the world’s best probe (Proposition 3); the funded factor of ten in precision deepens that wedge along the rows Table 2 names (Proposition 4); and the curvature it does not touch is a number, 10^{-12} to the field gradient and 10^{-21} to spacetime itself (Proposition 5). What would kill the paper’s central positioning is simple to state: a measurement showing the GQS spectrum reproduces a

combination another device already fixes, collapsing the coverage claim; or a systematics audit showing the wedge of Proposition 3 is an instrumental artifact of the kind that once haunted the COW phase. Absent those, the falling neutron remains what it has quietly been since 2002 — gravity’s only laboratory spectrum, waiting for the next factor of ten.

A Levels, mass combinations, and the tidal correction

This appendix derives Eq. (1) from the Schrödinger problem, works the algebra behind each row of Table 1, and carries the tidal correction of Proposition 5 to the closed form used in Eq. (5).

The eigenvalue problem. A neutron of inertial mass m_i and gravitational mass m_g above a mirror at $z = 0$ obeys

$$-\frac{\hbar^2}{2m_i} \psi''(z) + m_g g z \psi(z) = E_n \psi(z), \quad z > 0, \quad (6)$$

with $\psi(0) = 0$ at the mirror’s hard wall — the Fermi pseudopotential $V_F \approx 100$ neV [45] exceeds the peV eigenvalues by five orders and is idealized as infinite — and $\psi \rightarrow 0$ as $z \rightarrow \infty$. Introducing the gravitational length and energy

$$z_0 = \left(\frac{\hbar^2}{2m_i m_g g} \right)^{1/3}, \quad E_0 = m_g g z_0, \quad (7)$$

and the dimensionless variable $\zeta = z/z_0 - E_n/E_0$ recasts Eq. (6) as the Airy equation $\psi''(\zeta) = \zeta \psi(\zeta)$, whose decaying solution is $\psi \propto \text{Ai}(\zeta)$. The wall condition $\psi(0) = 0$ forces $\text{Ai}(-E_n/E_0) = 0$; since Ai vanishes at the arguments $-\varepsilon_n$ with $\varepsilon_n \in \{2.338, 4.088, 5.521, 6.787, \dots\}$, the eigenvalues are $E_n = \varepsilon_n E_0$. Substituting E_0 from Eq. (7),

$$E_n = \varepsilon_n m_g g \left(\frac{\hbar^2}{2m_i m_g g} \right)^{1/3} = \varepsilon_n \left(\frac{\hbar^2 m_g^2 g^2}{2m_i} \right)^{1/3}, \quad (8)$$

which is Eq. (1). Numerically $z_0 = 5.87 \mu\text{m}$ [59], $E_0 = 0.60$ peV, and $E_n = 1.41, 2.46, 3.32, 4.08$ peV for $n = 1-4$ [25], matching the transmission-resolved values 1.4, 2.5, 3.3, 4.1 peV [57]. The classical turning points $z_n = \varepsilon_n z_0$ are $\{13.7, 24.0, 32.4, 39.9, \dots\} \mu\text{m}$ [59].

The mass-combination table. Each device in Table 1 measures gravity through a different functional of (m_g, m_i) , and the algebra is elementary once the relevant observable is fixed. The bouncer eigenvalue, Eq. (8), gives

$$E_n \propto \left(\frac{m_g^2}{m_i} \right)^{1/3} g^{2/3}, \quad (9)$$

the GQS row. The spatial modulation of an eigenfunction in its classically allowed region has local wavevector $k(z) = \sqrt{2m_i(E_n - m_g g z)}/\hbar$, whose scale near the mirror is $k = z_0^{-1} = (2m_i m_g g/\hbar^2)^{1/3} \propto (m_g m_i)^{1/3}$, numerically $k \approx 3.3 \times 10^6 \text{ m}^{-1}$ for ^{87}Rb [48]. A freely falling or fountain trajectory obeys $\ddot{z} = -(m_g/m_i)g \equiv -\tilde{g}$, so any matter-wave fountain phase carries $\tilde{g} \propto m_g/m_i$, and $\Delta\phi = k\tilde{g}\tau^2$ [48]. A light-pulse atom interferometer instead imprints a phase $\sim k_{\text{eff}}gT^2$ with k_{eff} set by the laser, so its leading term is independent of the atomic mass and its Eötvös-ratio test isolates m_g/m_i [11]. The COW crystal interferometer’s gravitational phase scales as the product $m_g m_i$ — one power from the gravitational force $\propto m_g$ and one from the inertial dwell time (equivalently the de Broglie wavelength $\propto 1/m_i$) over the fixed crystal geometry — the textbook dependence whose laboratory and freely-falling-frame analyses are Greenberger and Overhauser’s [36, 48]. The five inequivalent functionals — $(m_g^2/m_i)^{1/3}$, $(m_g m_i)^{1/3}$, m_g/m_i , mass-free, and $m_g m_i$ — are the entries the integrator tabulates, with the GQS row alone measured on a bound spectrum.

The tidal correction in full. The uniform field is the leading term of the inverse-square field. Expanding $g(z) = g_0(1 + z/R_\oplus)^{-2}$ and integrating the force gives the potential energy

$$U(z) = m_g \int_0^z g(z') dz' = m_g g_0 \left(z - \frac{z^2}{R_\oplus} + \frac{z^3}{R_\oplus^2} - \dots \right), \quad (10)$$

so the leading correction to Eq. (6) is the tidal term $\delta V(z) = -m_g g_0 z^2/R_\oplus$. Its first-order shift needs the second moment of the Airy eigenstates. The linear-potential virial relation $2\langle T \rangle = \langle V \rangle$ fixes $\langle V \rangle = \frac{2}{3}E_n$, hence $\langle n|z|n \rangle = \frac{2}{3}\varepsilon_n z_0$, and the standard bouncer second moment is

$$\langle n|z^2|n \rangle = \frac{8}{15} \left(\frac{E_n}{m_g g} \right)^2 = \frac{8}{15} \varepsilon_n^2 z_0^2. \quad (11)$$

Therefore

$$\delta E_n = -\frac{m_g g_0}{R_\oplus} \langle n|z^2|n \rangle = -\frac{8}{15} \frac{\varepsilon_n^2 z_0^2 m_g g_0}{R_\oplus}, \quad \frac{\delta E_n}{E_n} = -\frac{8}{15} \frac{\varepsilon_n^2 z_0^2}{R_\oplus} \quad (12)$$

using $E_n = \varepsilon_n m_g g z_0$ and $g_0 \approx g$. For the ground state, where $\varepsilon_n = 2.338$, the values $z_0 = 5.87 \mu\text{m}$ and $R_\oplus = 6.37 \times 10^6 \text{ m}$ give $\delta E_n/E_n \approx -1.1 \times 10^{-12}$, the order quoted in Eq. (5). The correction is exactly the geodesic-deviation term: in Fermi normal coordinates the tidal potential is $\frac{1}{2}m_i c^2 R_{0z0z} z^2$ with $c^2 R_{0z0z} = \partial^2 \Phi / \partial z^2 = -2g_0/R_\oplus$ to Newtonian order, reproducing Eq. (12). Because that leading Riemann component equals the Newtonian second derivative, Eq. (12) is a bound already contained in Newton’s $1/r^2$ potential; the genuinely non-Newtonian, post-Newtonian part of R_{0z0z} is smaller by the surface potential $g_0 R_\oplus / c^2 \approx 6.9 \times 10^{-10}$,

so the fractional precision required to sense metric curvature rather than a non-uniform Newtonian field is $\sim (\varepsilon_n z_0 / R_\oplus)(g_0 R_\oplus / c^2) \approx 10^{-21}$, as stated in Proposition 5.

B Provenance of each exclusion curve

Figure 2 and Proposition 3 depend on putting incommensurable bounds on one (range, coupling) plane. This appendix records, curve by curve, the native quantity each experiment reports, the source it is drawn from, and the conversion to the common axes. Direct Yukawa points transfer exactly; the chameleon and symmetron conversions are model-dependent, and we grade them structured.

Torsion-balance Yukawa frontier (exact transfer). The Eöt-Wash and HUST pendulums report a Yukawa strength α at length λ in the potential $V(r) = -\alpha G m_1 m_2 e^{-r/\lambda}/r$, which is already the common axis. Eöt-Wash holds the inverse-square law to $|\alpha| \leq 1$ down to $\lambda = 56 \mu\text{m}$, bounds two large extra dimensions to $R \leq 44 \mu\text{m}$, and works below the dark-energy length $\lambda_d \approx 85 \mu\text{m}$ [49]; the successor run finds no deviation from $52 \mu\text{m}$ to 3.0 mm [53]; HUST reaches $|\alpha| \leq 1$ at $\lambda = 48 \mu\text{m}$ and improves the $40\text{--}350 \mu\text{m}$ band by up to a factor of three near $\lambda \approx 70 \mu\text{m}$ [73]. These define the plane’s macroscopic floor and the upper-range boundary of the wedge.

GQS Yukawa and spin-dependent bounds (exact transfer). Bouncer spectroscopy reports the same (α, λ) variables at shorter range: $\alpha < -8.5 \times 10^{10}$ (attractive) or $\alpha > 2.4 \times 10^{11}$ (repulsive) at $\lambda = 5.6 \mu\text{m}$ [25]. The large $|\alpha|$ reflects that Newtonian gravity between microscopic masses at microns is vanishingly weak, so a fifth force is bounded only as a large multiple of it — but at this range no torsion balance competes, which is the wedge. Spin-dependent couplings transfer likewise: the scalar–pseudoscalar product $g_s g_p > 3.7 \times 10^{-16}$ (attractive) or $> 5.3 \times 10^{-16}$ (repulsive) at $\lambda = 20 \mu\text{m}$ [45], plotted on the companion $(g_s g_p, \lambda)$ plane against polarized- ^3He limits spanning $100 \mu\text{m}$ to 1 cm that improved prior bounds by 10 to 30 [19].

Neutron-scattering closure (exact transfer, different probe). Reanalysis of neutron-scattering data bounds new short-range forces from 1 pm to 5 nm , several orders below prior limits [60], the picometer-scale reach that neutron-physics reviews single out [29]; pendellösung interferometry in perfect silicon adds an independent short-range neutron-nucleus bound over an overlapping range [39]. These occupy the plane’s short-range end and set the lower-range boundary of the wedge

— and they are scattering, not bouncing, so the neutron holds two disjoint bands of the plane by two different methods.

Chameleon conversion (model-dependent). A chameleon reports a matter coupling β whose effective range depends on the ambient density [50], so it is not natively a (α, λ) point. The conversion uses the field profile sourced by the mirror: Brax and Pignol solved that profile for the neutron bouncer, giving the level shift as a function of β and hence an equivalent short-range potential of range set by the profile’s thickness [15], with the parallel-plate and interferometric channels treated in the companion analysis [17]. Converted this way, the bouncer excludes $\beta < 6.9 \times 10^6$ at Ratra–Peebles index $n = 2$ [25] and $\beta > 5.8 \times 10^8$ for $2 \leq n \leq 4$ [45], plotted against the atom-interferometry chameleon wedge, which an in-vacuum source mass strengthened by over two orders of magnitude [37, 43, 68], its full nonlinear field solved on a grid to within 20% of the analytic estimate [30]. Because the range assigned to a β -bound depends on the density model, we mark these curves as model-converted on Figure 2.

Symmetron conversion (model-dependent). The symmetron lives in (M, λ, μ) [41]; its mirror-sourced profile was solved exactly, in Jacobi elliptic functions, for one- and two-mirror geometries, yielding a discrete set of solutions indexed by node number [16], which is the conversion from field parameters to a bouncer level shift. At the dark-energy-scale benchmark — vacuum mass $\approx 2.4 \times 10^{-3}$ eV, coupling scale ≈ 1 TeV — torsion pendulums exclude self-couplings $\lambda \lesssim 7.5$ [74], and the field’s spine review marks the “desert” of parameter space unconstrained between laboratory and astrophysical scales [18], part of which the micron band addresses; the collaboration’s own review situates the GQS and interferometry bounds together [71].

COW-class phases (a different coverage axis). The neutron-interferometry curves are drawn for context but belong to the $m_g m_i$ row, not the force plane (Section 2). The COW lineage verified the quantum equivalence of gravitational and inertial mass to about 1%, with a $\approx 1\%$ prediction-versus-measurement discrepancy surviving to a statistical precision of 1 part in 10^3 [20, 54], later attributed to a 10–40 nrad crystal misalignment [38]. We convert this not to a force bound but to a systematics cautionary marker: it is the record’s clearest case of an instrumental effect once read as an anomaly, and it sets the standard the wedge’s systematics budget must meet.

C The seeded computation

The level ladder, the transition frequencies, the exclusion wedge, and the projection come from one deterministic script, `sim_bounce.py`, whose source and outputs (`sim_bounce_output.txt/.json`) ship with the paper. There is no randomness; the seed recorded in the source (20260919) exists only for the record.

The ladder and its mass combination

With the neutron mass and g , the gravitational length is $z_0 = (\hbar^2/2m_i^2g)^{1/3} = 5.87 \mu\text{m}$ and the energy scale $E_0 = m_g g z_0 = 0.602$ peV, so the Airy levels $E_n = \varepsilon_n E_0$ run 1.407, 2.460, 3.321, 4.083, 4.780 peV for $n = 1\text{--}5$, with classical turning points $z_n = \varepsilon_n z_0$ from 13.7 to 47 μm (Fig. 1). The energy scale carries the mass combination $(m_g^2/m_i)^{1/3}$ — the writing $E_n = \varepsilon_n (\hbar^2 m_g^2 g^2 / 2m_i)^{1/3}$ makes it explicit — which is the axis Table 1 shows to be independent of the mass-free external-wave interferometer phase and the $m_g m_i$ product of the COW phase, and hence a genuine new column in the equivalence-principle coverage of the companion paper. The transition frequencies gravity-resonance spectroscopy reads are $(E_n - E_m)/h$: 255 Hz for $1 \rightarrow 2$, 463 Hz for $1 \rightarrow 3$, 184 Hz for $3 \rightarrow 4$, all in the hundreds-of-hertz band the qBounce experiments actually drive.

The wedge, the projection, and the scope

The script places the published screened-force bounds on common axes (Fig. 2): the qBounce chameleon bound $\beta < 5.8 \times 10^8$ at the micron scale, the symmetron islands, and the short-range Yukawa limits, against the torsion-balance frontier, exhibiting the $\lambda \sim 10^{-6}\text{--}10^{-5}$ m wedge where the neutron leads. The projection (Fig. 3, Table 2) carries the transition precision from today’s ~ 10 mHz to the ~ 1 mHz of the Ramsey–GRS upgrades and reads off the theory each factor of ten retires. Finally the scope check: the curvature (tidal) correction to a level is of order $z_0/R_\oplus \approx 9 \times 10^{-13}$ relative, so the system probes the Newtonian potential’s quantum mechanics, not space-time curvature, until far beyond current precision — the honest boundary Proposition 5 draws, and the reason the paper calls the falling neutron the oldest quantum-gravity experiment in the operational sense rather than the Planckian one.

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